2001-31242 서정관

1. delayed neutron의 생성 메커니즘.

① 핵분열 반응에 의해 2개의 방사 핵종, 수개의 즉발 중성자, 수개의 γ선이 생성된다.

② 이중 핵분열 파편인 방사핵종은 β^- 붕괴에 의해 안정화 되며, 어떤 핵종은 β^- 붕괴후 다시 중성자를 방출하고 안정화 된다. 이때 방출되는 중성자는 선행핵 붕괴속도에 따라 핵분열 반응후 0.1~60초 정도 지연되어 생성되므로 이를 지발 중성자라 한다.

2. 용어 설명

가. delayed neutron yield

$$\nu = \nu_p + \nu_d \quad (\text{neutron/fission})$$

where ν : average total number of neutrons

ν_p : prompt neutrons

ν_d : delayed neutrons

delayed neutron yield는 통상 아래와 같은 분율로 표현되며,

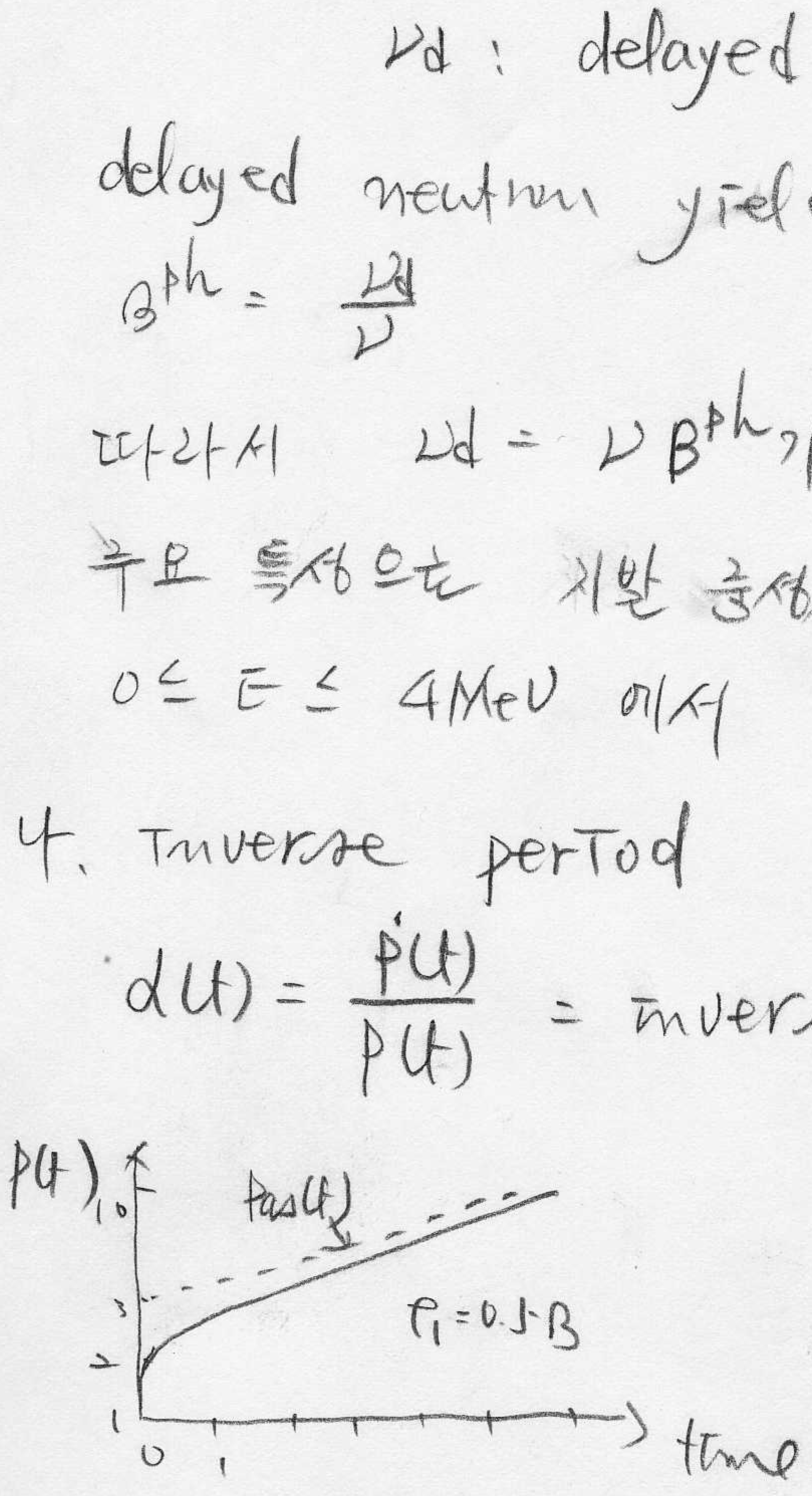
$$\beta^{ph} = \frac{\nu_d}{\nu}$$

따라서 $\nu_d = \nu \beta^{ph}$가 된다.

주요 특성은 지발 중성자의 대부분이 생성되는 energy 영역인 $0 \le E \le 4MeV$ 에서 ν_d = constant 하다.

4. inverse period

$$\alpha(t) = \frac{\dot{P}(t)}{P(t)} = \text{inverse period}.$$



반응도 삽입에 의한 출력 증강시 flux는 $P_{as}(t) = P_{as}\exp(\alpha t)$ 로 표시되고 왼쪽 그림과 같은 semi-log 그림에서 $\alpha(t)$는 flux 증가 기울기가 된다.

서정산

다. average neutron generation time

$$\Lambda = \frac{1}{\bar{v}\,\nu\Sigma_f}$$; average time between two birth events in successive generations.

라. mean free path

$$\frac{1}{\Sigma_f}$$; the average distance a neutron travels from its birth to a fission event.

마. average neutron lifetime

$$l = \frac{1}{\bar{v}}\frac{1}{\Sigma_a + DB^2}$$; 중성자가 생성되어 흡수 또는 누설될 때까지의 시간.

$$; \frac{\left(\text{the average traveling distance between birth and absorption or leakage } (1/(\Sigma_a + DB^2))\right)}{\bar{v}}$$

바. factorization vs. separation of the flux

- $p(t)$: a purely time dependent amplitude function
- $\psi(r, E, t)$; a space, energy, and time dependent shape function

① flux factorization ; $\phi(r, E, t) = p(t) \cdot \psi(r, E, t)$

중성자속은 time dependent amplitude ftn. 인 $p(t)$와 shape ftn. 인 $\psi(r, E, t)$로 변수 분리한 것, 가정 없음

② separation of the flux ; $\phi(r, E, t) = p(t) \cdot \psi(r, E)$

flux $\phi(r, E, t)$가 time dependent 항인 $p(t)$와 space, energy dependent 항인 $\psi(r, E)$로 분리가능하다고 가정한 것.

서정관

사. theoretically consistent flux shape corrections.

정적 반응도 측정 방식에 의해 동적 또는 정적 반응도가 계산된다. 그러나 일반적인 방식에 의해 동적 반응도가 계산되며, 두 결과는 theoretical conversion factor에 의해 서로 전환될 수 있다. 전환시 flux-shape correction factor에서 주요 차가 발생하며, 현재 λ mode flux가 가장 일반적으로 적용된다.

방식에서 생성되는 기본 양에는 ρ/β가 아니라 다음과 같이 정의된다.

$$\frac{\rho(t)}{\beta(t)} = \frac{(\Phi^{+}_{\lambda 0}, [F-M]\psi)}{(\Phi^{+}_{\lambda 0}, F_d \psi)}$$

*rod-drop experiment를 예로 설명하면,

- flux shape : $\psi_0(r,E) \xrightarrow{rapid} \psi_{PJ}(r,E,t) \xrightarrow{slow} \psi_{as}(r,E)$

where $\psi_{as}(r,E)$; desired quantity to be measured

$\psi_{PJ}(r,E,t)$: 방식에서는 ψ_{PJ}가 구해짐.

- the flux level or amplitude

$\frac{\rho_1}{\beta} = \frac{P_{PJ}-P_0}{P_{PJ}}$; flux level은 neutron counter로 측정됨.

$\Sigma^c(r,E)$를 space, energy dependent counter sensitivity라 하면

counting rate $CR(t) = \int_{counter}\int_E \Sigma^c(r,E)\,\phi(r,E,t)\,dE\,dV$

$$\frac{CR(t)}{CR_0} = \frac{P(t)}{P_0}\,\frac{R_\psi^c(t)}{R_\psi^c(0)}$$

the consistent amplitude function is obtained as a ratio of the measured and a specially calculated reaction rate.

$$P(t) = \frac{CR(t)}{R_\psi^c(t)}$$

서 창 완

3. reactivity measurements 방법 설명

가. subcriticality measurements by the rod-drop experiment.

rod drop으로 부반응도를 삽입하고 두 상태의 relative flux를 측정한다. 여기에서 unknown reactivity는 ρ_0 와 $\delta\rho_1$ 이다.

In a simplified analysis, the change in β and in the reduced sources are neglected. ($s_1 = s_0$, $\beta = \beta_0$)

$$P_0 = \frac{s_0}{-\rho_0} = \frac{\beta P_0 + s_0}{\beta - \rho_0} \quad -①$$

$$P_{PJ} = \frac{\beta P_0 + s_0}{\beta - \rho_1} \quad -②$$

$$P_1 = \frac{s_0}{-\rho_1} = \frac{\beta P_1 + s_0}{\beta - \rho_1} \quad -③$$

where $\rho_1 = \rho_0 + \delta\rho_1 \quad -④$

②/① : $\frac{P_{PJ}}{P_0} = \frac{\beta - \rho_0}{\beta - \rho_1} \quad -⑤$, ③/① ; $\frac{P_1}{P_0} = \frac{\rho_0}{\rho_1} \quad -⑥$

⑤. ⑥에서 ρ_1을 제거하면,

$$\frac{\rho_0}{\beta} = \frac{\left(\frac{1}{P_0} - \frac{1}{P_{PJ}}\right)}{\left(\frac{1}{P_1} - \frac{1}{P_{PJ}}\right)}$$

나. source- and Rod-Jerk Methods

① source-Jerk : 이 경우는 rod-drop과 마찬가지로 flux는 P_0에서 P_{PJ}로 내려간다.

이때 PJ식에서 $0 = (\rho_1 - \beta) P_{PJ} + \beta P_0 \quad -⑦$

$\rightarrow \frac{\rho_1}{\beta} = \frac{P_{PJ} - P_0}{P_{PJ}} \Rightarrow \frac{\rho_0}{\beta} = \frac{P_{PJ} - P_0}{P_{PJ}} < 0$ for initial reactivity

② rod-Jerk : 이 경우 source-Jerk와 부호만 반대로 된다.

즉 $\frac{\rho_1}{\beta} = \frac{P_{PJ} - P_0}{P_{PJ}} > 0$ (rod-Jerk out of a critical Rx)

서정란

4. delayed neutron source approximation 설명

가. one-delayed-group kinetics

지발중성자의 붕괴 상수를 $\bar{\lambda}$ 라는 평균값을 사용한다

$$\dot{p} = \frac{\rho-\beta}{\Lambda}p + \frac{1}{\Lambda}\bar{\lambda}\zeta(t)$$

이 경우 정확도는 ~1초 이하로 제한된다

나. precursor accumulation approximation

$$\dot{p} = \frac{\rho-\beta}{\Lambda}p + \frac{1}{\Lambda}[\beta p_0 + \bar{\lambda}\beta J(t)]$$

작은 시간 변화 동안 ζ는 p보다 늦게 변화하므로 $\zeta_k = \zeta_{k0}$로 둘 수 있다

새롭게 생성된 precursor는 축적되고 delayed neutron source 는 된다.

이러한 가정을 PA approximation 이라 한다. 위 식은 CDS approximation 의 first order extension 이며, sudden exchange 후 ~0.5초까지 적용된다.

다. constant delayed neutron source approximation

$$\dot{p} = \frac{\rho-\beta}{\Lambda}p + \frac{1}{\Lambda}\beta p_0$$

~0.1초 이하에서 $S_d(t) = S_{d0}$ 로 일정하다는 가정에서 사용한다

prompt jump 에 대해 정확하고 정상적인 해석을 가능하게 하는 최저 수준의 가정이다. sudden exchange 후 kinetics behavior 를 이해하는데 유용하다

라. prompt kinetics approximation

- delayed neutron source 무시

$$\dot{p} = \frac{\rho-\beta}{\Lambda}p$$

- superprompt-critical transients에 유용

서 정 산

5. one-group point kinetics equation 유도

- three simplification

(1) the time dependence of the flux $\phi(\underline{r},E,t)$ is assumed to be separable from its space and energy dependence

; $\phi(\underline{r},E,t) = p(t)\,\psi(\underline{r},E)$ — ①

(2) the local leakage loss ;

$D(\underline{r},E)\,B^2(\underline{r},E)\,\phi(\underline{r},E,t)$ — ②

(3) $\Sigma_f(\underline{r},E)$ is assumed independent of time

The shape function is set equal to the initial flux assuming a stationary state + $t<0$;

$\psi(\underline{r},E) = \phi_0(\underline{r},E)$ — ③

then the initial condition for $p(t)$;

$p(0) = p_0 = 1$ — ④

time-dependent diffusion equation ;

$$\frac{1}{v}\frac{\partial \phi(\underline{r},E,t)}{\partial t} = (\underline{F}_p - \underline{M})\,\phi(\underline{r},E,t) + S_d(\underline{r},E,t) + S(\underline{r},E,t) \quad ⑤$$

①, ②, ③, ④ → ⑤ and integrate with respect to energy ;

$$\int_0^\infty \frac{\psi(\underline{r},E)}{v(E)}\,dE\,\frac{dp}{dt} = \left[\int_0^\infty \nu_p\,\Sigma_f(\underline{r},E)\,\psi(\underline{r},E)\,dE - \int_0^\infty [\Sigma_a(\underline{r},E,t) + D(\underline{r},E)B^2(\underline{r},E)]\,\psi(\underline{r},E)\,dE\right] p(t)$$

$$+ \sum_k \lambda_k C_k(\underline{r},t) + \int_0^\infty S(\underline{r},E,t)\,dE \quad — ⑥$$

Integrating ⑥ with respect to space, and dividing by the integrated total shape function

$$\hat{\Psi} = \int_V \int_0^\infty \psi(\underline{r},E)\,dE\,dV \quad — ⑦$$

one-group equation is obtained:

$$\left(\overline{\frac{1}{v}}\right)\frac{dP}{dt} = [\nu_p\Sigma_f - \Sigma_a - DB^2]P + \frac{1}{\Psi}\sum_k \lambda_k \hat{C}_k + \frac{1}{\Psi}\hat{S} \quad -(8)$$

where $\left(\overline{\frac{1}{v}}\right) = \frac{1}{\Psi}\int_V\int_0^\infty \frac{\psi(\underline{r},E)}{v(E)}\,dE\,dV \quad -(9a)$

$$\nu_p\Sigma_f = \frac{1}{\Psi}\int_V\int_0^\infty \nu_p\Sigma_f(\underline{r},E)\,\psi(\underline{r},E)\,dE\,dV \quad -(9b)$$

$$\Sigma_a = \frac{1}{\Psi}\int_V\int_0^\infty \Sigma_a(\underline{r},E,t)\,\psi(\underline{r},E)\,dE\,dV \quad -(9c)$$

$$DB^2 = \frac{1}{\Psi}\int_V\int_0^\infty D(\underline{r},E)\,B^2(\underline{r},E)\,\psi(\underline{r},E)\,dE\,dV \quad -(9d)$$

$$\hat{C}_k(t) = \int_V C_k(\underline{r},t)\,dV \quad -(9e)$$

$$\hat{S}(t) = \int_V\int_0^\infty S(\underline{r},E,t)\,dE\,dV \quad -(9f)$$

Adding and subtracting $\nu_d\Sigma_f$ in the brackets of (8), and dividing the result by $\nu\Sigma_f$:

$$\Lambda\frac{dP}{dt} = (\rho-\beta)P + \frac{1}{\hat{S}_{f0}}\sum_k \lambda_k \hat{C}_k + \frac{\hat{S}}{\hat{S}_{f0}} \quad -(10)$$

where $\hat{S}_{f0} = \nu\Sigma_f\Psi_0 \quad -(11)$

$$\Lambda = \frac{1}{\nu\Sigma_f}\left(\overline{\frac{1}{v}}\right) \quad -(12)$$

To simplify the notation, the relative sources are introduced;

- relative neutron source $= \lambda_k\xi_k(t) = \lambda_k\frac{\hat{C}_k(t)}{\hat{S}_{f0}} \quad -(13)$

  where $\xi_k(t)$ = reduced precursors

- relative independent source $= s(t) = \frac{\hat{S}(t)}{\hat{S}_{f0}} \quad -(14)$

The balance equation for the precursor concentrations produced by all isotopes;

$$\frac{\partial C_k(\underline{r},t)}{\partial t} = -\lambda_k C_k(\underline{r},t) + \int_0^\infty \nu_{dk}\Sigma_f(\underline{r},E',t)\,\phi(\underline{r},E',t)\,dE' \quad -(15)$$

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Integrating 15 with respect to space and dividing by S_{F0};

$$\frac{d\xi_k(t)}{dt} = -\lambda_k \xi_k(t) + \beta_k p(t) \quad - \ 16$$

$$\text{where } \beta_k = \frac{\int_V \int_0^\infty \nu_{dk} \Sigma_f(\underline{r},E)\,\phi(\underline{r},E)\,dE\,dV}{\int_V \int_0^\infty \nu \Sigma_f(\underline{r},E)\,\phi(\underline{r},E)\,dE\,dV} \quad - \ 17$$

from 10 to 16 ;

* approximate point kinetics equation:

$$\frac{dp}{dt} = \frac{(\rho-\beta)}{\Lambda} p + \frac{1}{\Lambda}\sum_k \lambda_k \xi_k + \frac{1}{\Lambda} s(t)$$

$$\frac{d\xi_k(t)}{dt} = -\lambda_k \xi_k(t) + \beta_k p(t)$$

* familiar form;

point kinetics equation:

$$\frac{dp}{dt} = \frac{(\rho-\beta)}{\Lambda} p + \sum_k \lambda_k C_k + S_c$$

$$\frac{dC_k}{dt} = -\lambda_k C_k + \frac{\beta_k}{\Lambda} p$$

$$\text{where } C_k(t) = \frac{1}{\Lambda}\xi_k(t)$$

$$S_c(t) = \frac{1}{\Lambda} s(t)$$

6. delayed neutrons와 prompt neutrons 중 thermal reactor에서 importance가 더 큰 것:

$$r_{dk} = \frac{\int_E \chi_{dk}(E)\,\phi_0^*(E)\,dE}{\int_E \chi(E)\,\phi_0^*(E)\,dE}$$

In thermal reactor, the adjoint spectrum generally increase with decreasing energy because of increasing proximity to the range of the thermal fission.

서경선

fast fission is of minimal importance due to the rapid slowing down of the neutrons below the fast fission threshold in ^{238}U. Thus, the increase of $\Sigma_f \phi_0^*(E)$ with decreasing E prevails in the determination of the average importance of delayed and prompt fission neutrons. Therefore, delayed neutrons have a larger importance than prompt neutrons, in a thermal reactor.

17. point reactor model

; Point kinetics equation can be obtained from the exact point kinetics equations by introducing one major and two minor simplification

- The major simplification : neglecting the time dependence of the flux-shape

$$\psi(r, E, t) \simeq \phi_0(r, E) \quad -①$$

- The minor simplification: ⓐ the arbitrary denominator $F(t)$ is replaced by its initial value

$$F(t) \rightarrow F_0 = (\phi_0^*, F_0 \, \phi_0) \quad -②$$

ⓑ delayed fission neutron operator F_d is approximated by the initial operator

$$F_d \simeq F_{d0} \quad -③$$

서 한 샘

From these three simplification;

$$\rho(t) \to \rho^{(1)}(t) = \frac{1}{F_0}(\phi_0^*, [\Delta E - \Delta M]\psi_0) \quad -④$$

$$\Lambda(t) \to \Lambda_0 = \frac{k_0}{F_0} \quad -⑤$$

$$\frac{F_0}{F(t)} \to 1 \quad -⑥$$

$$\beta_k(t) \to \beta_{k0} = \frac{1}{F_0}(\phi_0^*, F_{dk0}\phi_0) \quad -⑦$$

from ④ to ⑦;

$$\begin{cases} \dot{p}(t) = \frac{\rho(t) - \beta}{\Lambda} p(t) + \frac{1}{\Lambda}\sum_k \lambda_k \xi_k(t) & -⑧ \\ \dot{\xi}_k(t) = -\lambda_k \xi_k(t) + \beta_k p(t) & -⑨ \end{cases}$$