RD Midterm 11/2/06

1. Rationale

a. After shutdown of a reactor, there exists still precursors which have to decay with a finite time constant (λ). Particularly the longest living precursor Br^{87} takes about 80 seconds to decay. Therefore the flux level due to delayed neutrons from the precursors maintains at least tens of seconds.

b. In fast reactors, the primary fuel nuclides are Plutonium isotopes which have much smaller delayed neutron fractions (~ 0.0025) than uranium isotopes (~ 0.0065). Moreover the neutron generation time (Λ) is two orders of magnitude smaller than LWRs. Because of the two reasons, namely lower β and smaller Λ, the transient response ~~in~~ a fast reactor is more sensitive to a reactivity insertion than in a LWR.

c. The prompt neutrons reacts right away to a step change in the reactor condition such as reactivity. The multiplication of prompt neutrons occurs promptly but the delayed neutrons need time for adjustment. Therefore there is a step change (jump) in the flux level, but the level stays constant until substantial change in delay neutron source occurs. This phenomenon is called prompt jump.

d. The one group decay constant is an average of the six group values. It could be either arimetically weighted average or harmonically weighted average. But in either case the average is inbetween the smallest and the largest values of λ_k, namely $\lambda_1 < \bar{\lambda} < \lambda_6$. The plot of inhour equation thus will look like the following for the two cases.

Therefore α_{1G}, the stable inverse period for one group is always larger than α_{6G}, the 6 group value for a positive reactivity insertion, which results in a faster increase of flux.

2. Physical significance

a. $\Lambda = \frac{1}{v}\frac{1}{v\Sigma_f}$ is the average time to take for the generation of a neutron.

b. $\beta_{eff} = \gamma \beta_{phy}$ is the total delayed neutron fraction augmented by the delayed neutron emission spectrum importance factor γ defined by

$$\gamma = \frac{\langle \varphi^*, \chi_d \rangle}{\langle \varphi^*, \chi \rangle}.$$

γ is about 1.05 for thermal reactors in which lower energy neutrons have larger importance. It reflects the lower energy of delayed neutrons (or the left shifted delayed neutron emission spectra)

c. Adjoint flux, $\varphi^*(r, E)$, is the importance of a neutron at r and E. Specifically it is the reactivity change per neutron absorption at r and E

d. $P = \dfrac{S_d + S}{\beta - \rho}$ is the generalized source multiplication formula which considers the delayed neutron source as an explicit source. The reactivity term in the denominator in this case has to be changed to $\beta - \rho$ from just $-\rho$ to account for the inclusion of delayed neutron into the source.

3. Derive

a. First order perturbation theory formula.

$$M_0 \phi_0 = \lambda_0 F_0 \phi_0$$
$$M_0^* \phi_0^* = \lambda_0 F_0^* \phi_0^*$$

$$(M_0 + \Delta M)(\phi_0 + \Delta\phi) = (\lambda_0 + \Delta\lambda)(F_0 + \Delta F)(\phi_0 + \Delta\phi)$$

0-th order : $M_0 \phi_0 = \lambda_0 F_0 \phi_0$

1-st order : $M_0 \Delta\phi + \Delta M \phi_0 = \lambda_0 (F_0 \Delta\phi + \Delta F \phi_0) + \Delta\lambda F_0 \phi_0$

Adjointing weighting on 1-st order terms

$$\langle \phi_0^*, M_0 \Delta\phi \rangle + \langle \phi_0^*, \Delta M \phi_0 \rangle = \lambda_0 \langle \phi_0^*, F_0 \Delta\phi \rangle + \lambda_0 \langle \phi_0^*, \Delta F \phi_0 \rangle + \Delta\lambda \langle \phi_0^*, F_0 \phi_0 \rangle$$

$$\cancel{\langle \phi_0^*, (M_0 - \lambda F_0) \Delta\phi \rangle} + \langle \phi_0^*, (\Delta M - \lambda_0 \Delta F) \phi_0 \rangle = \Delta\lambda \langle \phi_0^*, F_0 \phi_0 \rangle$$

$$\cancel{\langle \Delta\phi, (M_0^* - \lambda_0 F_0^*) \phi_0^* \rangle}$$

$$\Rightarrow \Delta\lambda = \frac{\langle \phi_0^*, (\Delta M - \lambda_0 \Delta F) \phi_0 \rangle}{\langle \phi_0^*, F_0 \phi_0 \rangle}$$

$$\Delta\rho = -\Delta\lambda = \frac{\langle \phi_0^*, (\lambda_0 \Delta F - \Delta M) \phi_0 \rangle}{\langle \phi_0^*, F_0 \phi_0 \rangle}$$

b. Inhour Equation

$$\Lambda \dot{p} = (\rho - \beta) p + \sum_k \lambda_k \zeta_k$$

$$\dot{\zeta}_k = -\lambda_k \zeta_k + \beta_k p$$

Let $\zeta_k = \tilde{\zeta}_k e^{\alpha t}$, $p = \tilde{p} e^{\alpha t}$

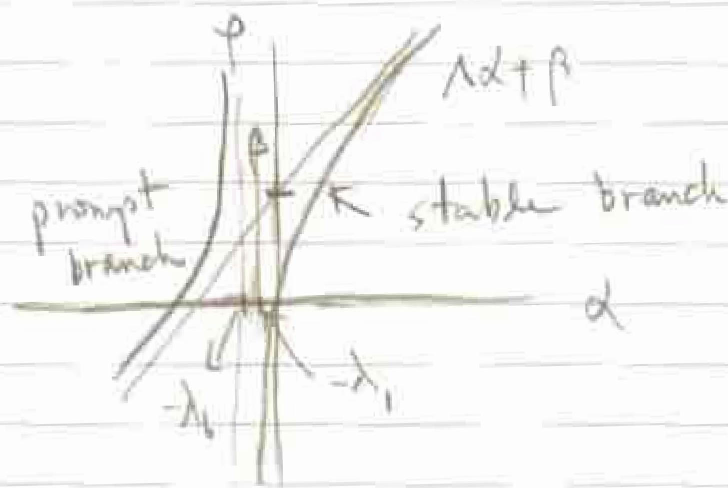
$$\Lambda \alpha \tilde{p} = (\rho - \beta) \tilde{p} + \sum_k \lambda_k \tilde{\zeta}_k \quad \cdots (1)$$

$$\alpha \tilde{\zeta}_k = -\lambda_k \tilde{\zeta}_k + \beta_k \tilde{p} \Rightarrow \tilde{\zeta}_k = \frac{\beta_k}{\alpha + \lambda_k} \tilde{p} \quad \cdots (2)$$

(2) in (1)

$$\Lambda \alpha \tilde{p} = (\rho - \beta) \tilde{p} + \sum_k \lambda_k \frac{\beta_k}{\alpha + \lambda_k} \tilde{p}$$

$$\rho = \Lambda \alpha + \beta - \sum_k \frac{\lambda_k \beta_k}{\alpha + \lambda_k} = \Lambda \alpha + \alpha \sum_k \frac{\beta_k}{\alpha + \lambda_k}$$



c. stable inverse period in one group

$$PJA \rightarrow \Lambda \dot{p} = 0$$

$$\Lambda \dot{p} = 0 = (\rho - \beta) p + \bar{\lambda} \zeta \Rightarrow \bar{\lambda} \zeta = (\beta - \rho) p$$

$$\dot{\zeta} = -\bar{\lambda} \zeta + \beta p \qquad \bar{\lambda} \dot{\zeta} = (\beta - \rho) \dot{p} \quad (\dot{\rho} = 0)$$

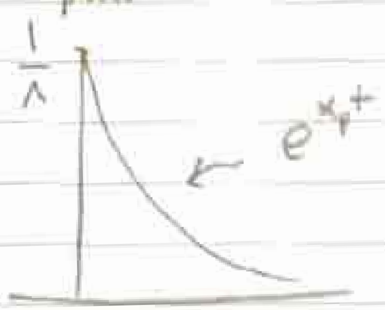
$$\bar{\lambda} \dot{\zeta} = -\bar{\lambda} (\beta - \rho) p + \bar{\lambda} \beta p$$
$$= \bar{\lambda} \rho p$$
$$= (\beta - \rho) \dot{p}$$

$$\Rightarrow \dot{p} = \frac{\bar{\lambda} \rho}{\beta - \rho} p \Rightarrow p(t) = p^0 e^{\frac{\bar{\lambda}\rho}{\beta - \rho} t} \quad \rightarrow \alpha_1$$

$$\therefore \alpha_1 = \frac{\bar{\lambda} \rho}{\beta - \rho}$$

d. Number of precursors generated per fission chain

For pulse



$$\dot{p} = \alpha_p p + \frac{1}{\Lambda} \delta(t), \quad \alpha_p = \frac{\rho - \beta}{\Lambda}$$

$$\frac{d}{dt}(p e^{-\alpha_p t}) = \frac{1}{\Lambda} e^{-\alpha_p t} \delta(t)$$

Integration

$$p e^{-\alpha_p t} - p_0 = \frac{1}{\Lambda} \qquad \int_0^t e^{-\alpha_p t'} \delta(t') dt' = 0$$

$$p(t) = \left(p_0 + \frac{1}{\Lambda}\right) e^{\alpha_p t}$$

$$\frac{1}{\Lambda}\int_0^\infty e^{\alpha_1 t}\,dt = \frac{1}{\Lambda}\,\frac{-1}{\alpha_p} = \frac{1}{\beta-\rho}$$

$\Rightarrow \frac{1}{\beta-\rho}$ fission neutrons would result from a fission chain. Thus the precursor to be generated in a fission chain is $\frac{\beta}{\beta-\rho}$.

4. $$\frac{1}{v}\frac{\partial\phi}{\partial t} = (F_1 - F_d)\phi - M\phi + \sum_k \chi_{dk}\lambda_k C_k$$

i) Factorization

Let $\phi(r,E,t) = p(t)\,\psi(r,E,t)$

choose $p(t)$ to carry the rapid variation part of the flux level and $\psi(r,E,t)$ to reflect the gradual change in flux shape

Then LHS

$$\frac{1}{v}\frac{\partial\phi}{\partial t} = \frac{1}{v}\left(\dot{p}\,\psi + p\,\frac{\partial\psi}{\partial t}\right)$$

ii) Adjoint weighting, Let $M\phi_0^* = \lambda_0 F_0 \phi_0^*$

$$\langle \phi_0^*, \frac{1}{v}\frac{\partial\phi}{\partial t}\rangle = \langle \phi_0^*, \frac{1}{v}(\dot{p}\psi + p\frac{\partial\psi}{\partial t})\rangle$$

$$= \dot{p}\langle \phi_0^*, \frac{1}{v}\psi\rangle + p\frac{\partial}{\partial t}\langle \phi_0^*, \frac{1}{v}\psi\rangle$$

The adjoint weighting is performed to account for the different importance of delayed neutrons as well as the different importance in different location (r) and energy (E).

RHS first two terms

$$\langle \phi_0^*, (F - M - F_d)\psi \rangle p$$

iii) Constraint

Let $\langle \phi_0^*, \frac{1}{v}\psi \rangle = K$
This constraint makes the factorization consisting of two functions unique

iv) Divide by $F(t) = \langle \phi_0^*, F\phi \rangle$

LHS

$$\dot{p}\frac{\langle \phi_0^*, \frac{1}{v}\psi \rangle}{\langle \phi_0^*, F\phi \rangle} = \Lambda \dot{p} \quad \text{where} \quad \Lambda = \frac{\langle \phi_0^*, \frac{1}{v}\psi \rangle}{\langle \phi_0^*, F\phi \rangle}$$

RHS

$$\left(\frac{\langle \phi_0^*, (F-M)\psi \rangle}{\langle \phi_0^*, F\phi \rangle} - \frac{\langle \phi_0^*, F_d\psi \rangle}{\langle \phi_0^*, F\phi \rangle} \right) p$$

$$= \left(\rho(t) - \beta(t) \right) p(t)$$

where $\rho(t) = \dfrac{\langle \phi_0^*, (F-M)\psi \rangle}{\langle \phi_0^*, F\phi \rangle}$

Point Reactor Model

① $\psi(r, E, t) = \psi_0(r, E)$: Neglect the change in shape

② $F(t) = F_0(t)$, $\beta(t) = \beta_0$, $\Lambda(t) = \Lambda_0$

5. Simplification of In-hour Equation

$$\rho = \Lambda\alpha + \beta - \sum_k \frac{\lambda_k \beta_k}{\alpha + \lambda_k}$$

i) $\alpha \gg \lambda_k$

neglect 2-nd order in $\frac{\lambda_k}{\alpha}$

$$\sum_k \frac{\lambda_k \beta_k}{\alpha+\lambda_k} = \sum_k \frac{\lambda_k}{\alpha}\beta_k \left(\frac{1}{1+\frac{\lambda_k}{\alpha}}\right) \simeq \sum_k \frac{\lambda_k}{\alpha}\beta_k \left(1-\frac{\lambda_k}{\alpha}\right) \simeq \sum_k \frac{\lambda_k}{\alpha}\beta_k$$

$$= \frac{1}{\alpha}\bar{\lambda}\beta \quad \text{where} \quad \bar{\lambda} = \frac{\sum_k \lambda_k \beta_k}{\beta}$$: delayed neutron weighted λ

$$\therefore \rho = \Lambda\alpha + \beta - \frac{1}{\alpha}\bar{\lambda}\beta$$

For 1-G $\quad \rho = \Lambda\alpha + \beta - \frac{\lambda\beta}{\alpha+\lambda} \longrightarrow \rho = \Lambda\alpha + \beta - \frac{\lambda\beta}{\alpha}$

if $\lambda \ll \alpha$

$$\therefore \lambda = \bar{\lambda}$$

ii) $\alpha \ll \lambda_k$

$$\sum_k \frac{\lambda_k \beta_k}{\alpha+\lambda_k} = \sum_k \frac{\beta_k}{\left(1+\frac{\alpha}{\lambda_k}\right)} \simeq \sum_k \beta_k\left(1-\frac{\alpha}{\lambda_k}\right) = \beta - \sum \frac{\beta_k}{\lambda_k}\alpha$$

$$= \beta - \bar{\lambda}^{m}\alpha\beta \quad \text{where} \quad \frac{1}{\bar{\lambda}^{m}} = \frac{\sum_k \frac{1}{\lambda_k}\beta_k}{\beta}$$

$\therefore \rho = \Lambda\alpha + \beta - \beta + \bar{\lambda}^{in}\alpha = \Lambda\alpha + \bar{\lambda}^{in}\alpha$

For 1-G

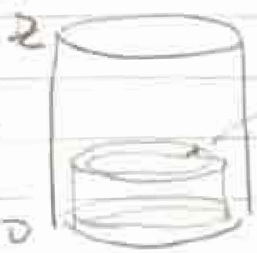
$$\rho = \Lambda\alpha + \beta - \frac{\lambda\beta}{\alpha+\lambda} = \Lambda\alpha + \beta - \frac{\beta}{1+\frac{\alpha}{\lambda}} \doteq \Lambda\alpha + \beta - \beta\left(1-\frac{\alpha}{\lambda}\right)$$

$$= \Lambda\alpha + \frac{\beta}{\lambda}\alpha$$

$\therefore \lambda = \bar{\lambda}^{in}$

6.

a.



First order perturbation Formula

$$\Delta\rho = \frac{\langle \phi_0^*, \lambda(\Delta F - \Delta M)\phi \rangle}{\langle \phi_0^*, F\phi \rangle}$$

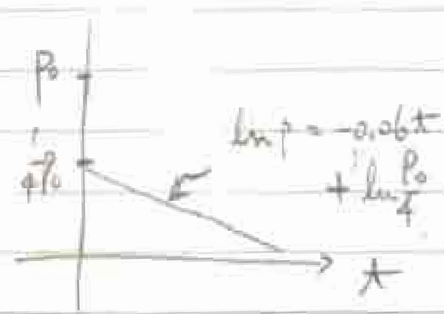
$$\phi_0 = \tilde{\phi}\, J_0\left(\frac{2.405}{R}r\right) \sin\left(\frac{\pi}{H}z\right)$$

$$\phi_0^* = \phi \quad , \quad \Delta\Sigma_f = 0$$

$$\Delta\rho = \frac{\langle \phi_0^*, -\lambda_0\Delta\Sigma_a\phi \rangle}{\langle \phi_0^*, \nu\Sigma_f\phi \rangle} = \frac{-\lambda_0\int_0^z\int_0^R \Delta\Sigma_a(r,z) J_0^2\left(\frac{2.405}{R}r\right)\sin^2\left(\frac{\pi}{H}z\right)2\pi r\,dr\,dz}{\nu\Sigma_f\int_0^z\int_0^R J_0^2\left(\frac{2.405}{R}r\right)\sin^2\left(\frac{\pi}{H}z\right)2\pi r\,dr\,dz}$$

b. $P_{PJ} = \dfrac{\beta}{\beta-\rho_1}P_0 \qquad \rho_1 = -3\,\$$

$$f_{PJ} = \frac{1}{1-\frac{\rho_1}{\beta}} = \frac{1}{1+3} = \frac{1}{4}$$



$\bar{\lambda}^{in}$

$$\alpha_1 = \frac{\lambda\rho_1}{\beta-\rho_1} = \frac{0.08\times(-3)}{1+3} = 0.02(-3) = \underline{-0.06}$$