Four

STATIC PERTURBATION THEORY

4-1 The Basic Reactor Eigenvalue Problem and the Perturbation Theory App.

4-1A Motivations for a Perturbation Theory

* Correct treatment of complicated configuration (three-dimensional configurations with spatially varying heterogeneous material composition)
  → tedious & expensive

* So we need proper approximations with sufficient accuracy

* The complications that have been eliminated in these approximate solutions can then be treated as "perturbations" of simplified cases.

* The application of perturbation theory is very advantageous for the calculation of the perturbation of multiplication constants k (= *keff*) for complicated configurations.

* Especially when the calculation of the corresponding perturbed neutron flux is not required. → Static perturbation theory frequently and commonly applied.

4-1B Neutron Multiplication as an Eigenvalue Problems

* the concept of the multiplication constant, k → an eigenvalue in a static neutron balance eq.

$$\boldsymbol{M\Phi} = \lambda \boldsymbol{F\Phi} \tag{4.1}$$

[(migraton & loss of neutrons)=(modified source of fission neutrons)]

multiplication constant, k defined (by eigenvalue λ)

$$k = \frac{1}{\lambda} \tag{4.2}$$

* For a simple one-group model,

  Migration and loss operator, $\boldsymbol{M} = DB^2 + \Sigma_a$ (4.5a)

  Fission source operator, $\boldsymbol{F} = \nu\Sigma_f$ (4.5b)

  $\rightarrow (DB^2 + \Sigma_a)\hat{\Phi} = \lambda\nu\Sigma_f\hat{\Phi}$ (4.6)

  $\frac{1}{\lambda} = k = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a}$ (=1, specially critical) (4.7)

* If the reactor is not critical ($\nu\Sigma_f - (DB^2 + \Sigma_a) \neq 0$), the flux must be either time dependent or equal to zero

* Reactivity (static reactivity), ρ ; the degree of off-criticality

  $1 - \lambda = 1 - \frac{1}{k} = \frac{\Delta k}{k} = \rho$ (4.11)

* Solution of the eigenvalue problem, Eq.(4.6) can be found by solving a simple algebraic equation (matrix form, application of direct inversion or iterative method)

4-1C The Basic Approach for the Calculation of an Eigenvalue Perturbation

* Perturbed & Unperturbed system

  1) perturbed system : more complicated actual system

  $\boldsymbol{M\Phi} = \lambda\boldsymbol{F\Phi}$ (4.1)

  2) unperturbed system : clean system

  $\boldsymbol{M}_o\boldsymbol{\Phi}_o = \lambda_o\boldsymbol{F}_o\boldsymbol{\Phi}_o$ (4.12)

* Expression of perturbed quantities

  $\Delta\lambda = \lambda - \lambda_o = -\frac{\Delta k}{k_0 k} = -\Delta\rho$ (4.13)

  where $-\Delta\rho$ is "reactivity increment," and $\Delta k = k - k_0$ (4.14)

  $\Delta\boldsymbol{M} = \boldsymbol{M} - \boldsymbol{M}_o$ (4.15a)

$$\Delta \boldsymbol{F} = \boldsymbol{F} - \boldsymbol{F}_o \qquad (4.15b)$$

$$\Phi = \Phi_o + \Delta\Phi \qquad (4.18)$$

* Mathematical formulation

$$\boldsymbol{A}\Phi = \lambda\Phi \qquad (4.17a)$$

$$\boldsymbol{A_0}\Phi_0 = \lambda_0\Phi_0 \qquad (4.17b)$$

* Introduce Φ_0 (Eq. (4.18)) into Eq. (4.17a)

$$\boldsymbol{A}(\Phi_0 + \Delta\Phi) = \lambda(\Phi_0 + \Delta\Phi)$$

$$\therefore \boldsymbol{A}\Phi_0 = \lambda\Phi_0 - (\boldsymbol{A}\Delta\Phi - \lambda\Delta\Phi) \qquad (4.19)$$

* These equations depend on space and energy. Since the desired quantity, $\Delta\lambda$, is just a number, the space and energy dependencies are removed by integration. To preserve generality and to add flexibility to the procedure, the equations are multiplied with a *weighting function,* $\Phi^w = \phi^w(\boldsymbol{r}, E)$, prior to the integration.

* Multiplication of Eqs. (4.19) & (4.17b) with a weighting function,(see Sec. B-1 of App.B, "scalar product")

$$(\Phi^w, \boldsymbol{A}\Phi_0) = \lambda(\Phi^w, \Phi_0) - (\Phi^w, [\boldsymbol{A} - \lambda]\Delta\Phi) \qquad (4.20a)$$

$$(\Phi^w, \boldsymbol{A}_0\Phi_0) = \lambda_0(\Phi^w, \Phi_0) \qquad (4.20b)$$

* Subtracting Eqs (4.20a) and (4.20b),

$$(\Phi^w, \Delta A\Phi_0) = \Delta\lambda(\Phi^w, \Phi_0) - (\Phi^w, [A - \lambda]\Delta\Phi) \qquad (4.21)$$

* The second term on the right side of Eq. (4.21),

$$(\Phi^w, [\boldsymbol{A} - \lambda]\Delta\Phi) = (\Phi^w, [\boldsymbol{A_0} - \lambda_0]\Delta\Phi) + (\Phi^w, [\Delta\boldsymbol{A} - \Delta\lambda]\Delta\Phi) \qquad (4.23)$$

* If the *difference of the perturbed and unperturbed systems is small*, the second-order term in Eq. (4.23) is small compared to the first-order term.

* If the first-order term can be eliminated, the right side of Eq.(4.23) *reduced to only a small second-order term*.

* If Φ^w *is chosen to be the adjoint function* Φ_0^w (the solution of the adjoint eigenvalue problem) ⇒ the first term on the right side of Eq. (4.23) can be eliminated. -> *Eq.(4.23) is neglected*.

* Therefore, Eq. (4,21)

$$(\Phi^w, \Delta A \Phi_0) = \Delta\lambda (\Phi^w, \Phi_0)$$

$$\therefore \Delta\lambda \cong \frac{(\Phi_0^*, \Delta A \Phi_0)}{(\Phi_0^*, \Phi_0)} = \Delta\lambda^{(1)} \qquad (4.26)$$

where, $\Delta\lambda^{(1)}$ is the result of "first-order perturbation theory".

4-2 First-Order Perturbation Theory

* The application of the same approach (4-1C) to the eigenvalue problem of reactor (some additional terms)

* Derivation of First-order perturbation theory

1) The perturbed & unperturbed problem (neutron balance equation)

$$M\Phi = \lambda F\Phi \qquad (4.27)$$

$$M_o\Phi_o = \lambda_o F_o \Phi_o \qquad (4.28)$$

2) Decomposing the flux , insert Eq.(4.18) into Eq.(4.27)

$$M\Phi_o = \lambda F\Phi_o - (M - \lambda F)\Delta\Phi \qquad (4.29a)$$

3) The first term on the right side Eq. (4.29a) is recast into zero-, 1st-, and 2nd -order terms :

$$\lambda F\Phi_o = \lambda F_o\Phi_o + \lambda_o \Delta F\Phi_o + \Delta\lambda\Delta F\Phi_o \qquad (4.30)$$

$$= \lambda F_o\Phi_o + \lambda_o \Delta F\Phi_o \text{ (neglecting the 2nd-order term)}$$

4) Insert Eq.(4.30) into Eq.(4.29)

$$M\Phi_o = \lambda F_o\Phi_o + \lambda_o \Delta F\Phi_o - (M - \lambda F)\Delta\Phi \qquad (4.29b)$$

5) Multiplying Eq.(4.29b) and Eq.(4.28) with a weighting function (Φ^w)

$$(\Phi^w, \boldsymbol{M}\Phi_o) = \lambda(\Phi^w, \boldsymbol{F}_o\Phi_o) + \lambda_o(\Phi^w, \Delta\boldsymbol{F}\Phi_o) - (\Phi^w, [\boldsymbol{M} - \lambda\boldsymbol{F}]\Delta\Phi) \quad (4.31a)$$

$$(\Phi^w, \boldsymbol{M}_o\Phi_o) = \lambda_o(\Phi^w, \boldsymbol{F}_o\Phi_o) \quad (4.31b)$$

6) Subtracting Eq.(4.31a) & Eq.(4.31b)

$$(\Phi^w, [\Delta\boldsymbol{M} - \lambda_o\Delta\boldsymbol{F}]\Phi_o) = \Delta\lambda(\Phi^w, \boldsymbol{F}_o\Phi_o) - (\Phi^w, [\boldsymbol{M} - \lambda\boldsymbol{F}]\Delta\Phi) \quad (4.32)$$

7) The term containing the flux deformation ($\Delta\Phi$) can be eliminated in a 1st-order approximation.

$$\begin{aligned}(\Phi^w, [\boldsymbol{M} - \lambda\boldsymbol{F}]\Delta\Phi) = \; & (\Phi^w, [\boldsymbol{M}_o - \lambda_o\boldsymbol{F}_o]\Delta\Phi) \\ & + (\Phi^w, [\Delta\boldsymbol{M} - \lambda_o\Delta\boldsymbol{F} - \Delta\lambda\boldsymbol{F}_o]\Delta\Phi) \\ & - \Delta\lambda(\Phi^w, \Delta\boldsymbol{F}\Delta\Phi) \\ = \; & (\Phi^w, [\boldsymbol{M}_o - \lambda_o\boldsymbol{F}_o]\Delta\Phi) \\ & \textit{(neglect the 2nd- and 3rd-order term)}\end{aligned} \quad (4.33)$$

8) Then, choose the unperturbed adjoint flux Φ_o^* as the weighting function Φ^w

$$\begin{aligned}(\Phi^w, [\boldsymbol{M}_o - \lambda_o\boldsymbol{F}_o]\Delta\Phi) &= (\Delta\Phi, [\boldsymbol{M}_o^* - \lambda_o\boldsymbol{F}_o^*]\Phi^w) \\ &= (\Delta\Phi, [\boldsymbol{M}_o^* - \lambda_o\boldsymbol{F}_o^*]\Phi_o^*) \\ &= 0\end{aligned} \quad (4.34)$$

(since the adjoint flux Φ_o^* is the solution of the adjoint eigenvalue problem)

9) Remainder of Eq.(4.32)

$$(\Phi^w, [\Delta\boldsymbol{M} - \lambda_o\Delta\boldsymbol{F}]\Phi_o) = \Delta\lambda(\Phi^w, \boldsymbol{F}_o\Phi_o) \quad (4.32')$$

* The First order perturbation formula for reactivity increment

$$\Delta\rho = -\frac{(\Phi_o^*, [\lambda_o \Delta \boldsymbol{F} - \Delta \boldsymbol{M}]\Phi_o)}{(\Phi_o^*, \boldsymbol{F}_o \Phi_o)} = -\Delta\lambda \tag{4.36}$$

$$= -\frac{\int_V \int_E \Phi_o^*(\boldsymbol{r}, E)(\lambda_o \Delta \boldsymbol{F} - \Delta \boldsymbol{M})\Phi_o(\boldsymbol{r}, E)\, dE\, dV}{\int_V \int_E \Phi_o^*(\boldsymbol{r}, E), \boldsymbol{F}_o \Phi_o(\boldsymbol{r}, E) dE dV}$$

* The first-order perturbation theory formulas are called "stationary" with respect to amall changes in the system because the error of the first-order reactivity evolves only "quadratically" with a linearly increasing change.