

$\frac{h}{2m}$ $\frac{h}{2m} = \frac{h}{2m} = \frac{h}{2m}$ $\frac{1}{2}mv^2 = mc^2$

(Note) Select and give only the answer to questions 1 – 12. For questions 13 – 15, give the answer in sentence(s) with your explanation. Answers can be either in Korean or in English or in both mixed.

1. Which is the name of process for a photon scattering incoherently from a free electron and changing its wavelength after the scattering? (PC2-4)

- ① Thomson scattering
- ② Compton scattering
- ③ Rutherford scattering
- ④ de Broglie scattering

2. What is the kinetic energy of an electron to have the de Broglie wavelength of 1 Å? Choose the closest answer. (Hint: $hc = 197 \text{ MeV}\cdot\text{fm}$, electron mass $m_e = 511 \text{ keV}/c^2$) (PC2-4)

- ① 150 eV
 - ② 1.5 keV
 - ③ 15 keV
 - ④ 150 keV
- $\lambda = \frac{h}{mv} = \frac{hc}{mc^2} \lambda = \frac{hc}{E}$ $E = \frac{hc}{\lambda} = \frac{197 \text{ MeV}\cdot\text{fm}}{10^{-10} \text{ m}} = 1970 \text{ eV}$

3. For some composite wave, the relation between the frequency ω and the wave number k leads to a propagation speed of $d\omega/dk$ depending on the wave number of the constituent wave. What is the propagation speed of $d\omega/dk$ called by? (PC2-4)

- ① phase velocity
- ② group velocity
- ③ dispersion medium
- ④ terminal speed

4. Which is the time-dependency of the wave of a particle in a steady state of energy E ? (PC1-1)

- ① $\exp(-iEt/\hbar)$
- ② $\exp(-Et/\hbar)$
- ③ $\sin(kx - \omega t)$
- ④ $\Psi(x, t)$

5. Consider two arbitrary functions ψ_1 and ψ_2 in Hilbert space, both vanishing in the boundary region of the integrable space.

When the condition $\int \psi_1^* \hat{H} \psi_2 d\tau = \int (\hat{H} \psi_1)^* \psi_2 d\tau$ is met for an operator \hat{H} , what is the operator \hat{H} called? (PC1-1)

- ① Hilbert operator
- ② Hamiltonian operator
- ③ Hermitian operator
- ④ Heisenberg operator

6. Choose the wrong one in the following statements about the boundary conditions on the Schrödinger wave function $\psi(x)$ at positions of changing behaviour for the potential function $V(x)$. (PC5-1)

- ① Where the potential goes to $+\infty$ or $-\infty$ at some definite point, the $\psi(x)$ is continuous but $d\psi(x)/dx$ is not continuous at the point.
- ② In the region where the potential is finite, the $\psi(x)$ and $d\psi(x)/dx$ are both continuous.
- ③ If the potential is discontinuous at some definite point while it remains finite, the $\psi(x)$ is continuous but $d\psi(x)/dx$ is not continuous at that point.
- ④ Where there is continuity in $V(x)$, there are also continuities in $\psi(x)$ and $d\psi(x)/dx$.

7. Choose the equality of the quantum commutator bracket $[x, \hat{p}_x]$? (PC5-1)

- ① \hbar
- ② 0
- ③ $-\hbar$
- ④ $\hbar p_z$

8. How many possibilities for each of m_l and m_s of an electron in d-orbital ($l=2$)? (Note: m_l and m_s are the magnetic quantum number for the orbital and the intrinsic spin angular momentum, respectively.) (PC2-5)

- ① 2, 0
- ② 1, 3
- ③ 5, 2
- ④ 3, 1/2

(continued)
 $l=3$
 $l=2$
 m
 $s \rightarrow p \rightarrow d$

9. For one dimensional bound state motion in the following potentials, which allows its wave function to have a definite parity? (Note: $\beta, V_0, a, k > 0$) (PC5-1)

① $V(x) = \frac{1}{2} \beta x^2$ ✓

② $V(x) = \begin{cases} -V_0, & |x| \leq a \\ 0, & x < -a \\ +V_0, & x > a \end{cases}$ ✗

③ $V(x) = \begin{cases} -V_0 \sin \frac{\pi}{a} x, & |x| \leq a \\ 0, & \text{otherwise} \end{cases}$ ✓

④ $V(x) = -\frac{V_0}{x}$ ✗

10. Select an inadequate description about the x-rays generated in a typical electron tube of 100 kV maximum. (PC2-5)

- ① Most of the electron's energy is converted to x-ray photons. ○
- ② Most of the x-rays are the Bremsstrahlung photons and their wavelength distribution is continuous. ✓
- ③ The characteristic x-ray of target atom can be emitted if the electron energy is sufficient. ✗
- ④ There is a minimum wavelength for the emitted x-rays. ○

11. Which is a proper unit for the mass attenuation coefficient μ_m of a material? (PC2-5)

- ① barn
 - ② cm^{-1} ✓
 - ③ cm^2/g ✓
 - ④ $\text{g} \cdot \text{barn}/\text{cm}^3$
- $MP = 3$

12. For $n = 4$ shell (n : principal quantum number ≥ 1) in the hydrogen atom, how many l -orbitals are permitted? (PC2-5)

- ① 1
 - ② 2
 - ③ 3 ✓
 - ④ 4 ✓
- $\frac{h^2}{2m} \frac{d^2 \psi}{dx^2} + V(x) \psi(x) = E \psi(x)$

13. The spatial wave function $\psi(x)$ is given by $\psi(x) = \sqrt{2/a} \sin \frac{\pi}{a} x, |x| \leq a/2; \psi(x) = 0, \text{ otherwise}$.

What is the expectation value of the squared momentum p_x^2 ? (PC5-1)

$$\langle p_x^2 \rangle = \int_{-a/2}^{a/2} \psi^*(x) p_x^2 \psi(x) dx = \int_{-a/2}^{a/2} \psi^*(x) \left(-\hbar^2 \frac{d^2 \psi}{dx^2} \right) dx = \int_{-a/2}^{a/2} \psi^*(x) \sqrt{2/a} \sin \frac{\pi}{a} x \left(\hbar^2 k^2 \right) \left(-\sin \frac{\pi}{a} x \right) dx = \sqrt{2/a} \hbar^2 k^2 \int_{-a/2}^{a/2} \sin^2 \frac{\pi}{a} x dx = \sqrt{2/a} \hbar^2 k^2 \left(\frac{a}{2} \right) = \sqrt{2/a} \hbar^2 k^2 \frac{a}{2} = \sqrt{2/a} \hbar^2 k^2 \frac{a}{2}$$

14. By considering the spherical harmonic function $Y_{lm}(\theta, \phi)$, answer to the following questions: (PC1-1)

(a) What is the parity of $Y_{lm}(\theta, \phi)$? How is it decided? (Note: $Y_{lm}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_m(\cos \theta) \exp(im\phi)$,

$$P_m(\cos \theta) = \frac{(-1)^m}{2^l l!} (1 - \cos^2 \theta)^{l/2} \frac{d^{l+m}}{d(\cos \theta)^{l+m}} (\cos^2 \theta - 1)^l$$

(b) Show that $Y_{21}(\theta, \phi) = \sqrt{15/8\pi} \cos \theta \sin \theta \exp(+i\phi)$ is a normalized function in (θ, ϕ) coordinate space.

15. (a) List three examples or application problems which are understood or solved by using the Schrödinger wave equation. (PC1-1)

(b) List three examples of x-ray applications which are already realized or can be feasible. In the latter case, explain your idea about the feasibility. (PC4-1)

Ionization chamber
Fluorescence