

2006학년도 2학기

응용핵물리

중간고사 문제지

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$$\frac{1}{226} \times 6.02 \times 10^{23} \times \frac{1 \text{ h}}{3600 \times 24 \times 365}$$

§ 문 1-8 (총 48점)까지는 답 만을, 문 9-12 (총 60점)는 풀이과정과 답을 작성하시오.

1. The half-life of ^{226}Ra is 1620 y. What is the activity of 1 g of ^{226}Ra ? (Atomic mass of $^{226}\text{Ra} = 226.0254 \text{ u}$)

- ① $3.7 \times 10^{10} \text{ Bq}$ ② 0.1 Ci ③ $2.26 \times 10^9 \text{ Bq}$ ④ 2.26 Ci

2. 자연계의 붕소(B)는 ^{10}B 와(과) ^{11}B 동위원소의 혼합이다. 붕소의 원자량은 10.811u이며 ^{10}B 의 atomic mass 10.013 u, ^{11}B 의 atomic mass 11.009u이다. 자연계 붕소 내의 ^{10}B 의 존재비(abundance)로 가장 가까운 것은?

- ① 10% ② 15% ③ 20% ④ 25%

$$10.811 = 10.013 \times \frac{100-x}{100} + 11.009 \times \frac{x}{100}$$

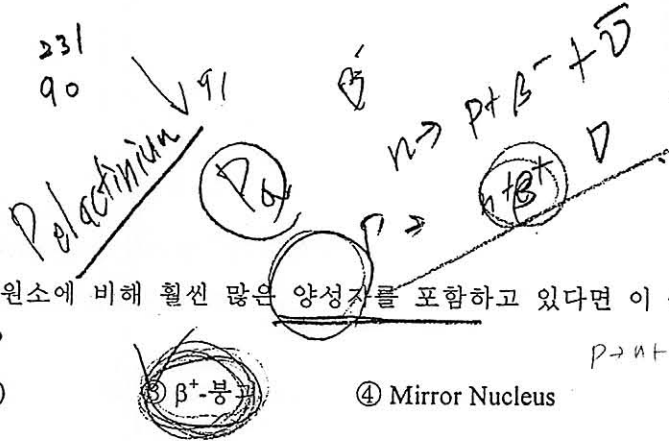
3. 다음 중 γ -붕괴와 경쟁적으로 일어날 수 있는 것은?

- ① α -붕괴 ② β^- -붕괴 ③ EC ④ IC

4. $^{235}_{92}\text{U}$ 는 한번의 α -붕괴와 한번의 β^- -붕괴를 거쳐 $^{231}_{91}\text{Pa}$ 로 변환된다. 여기서 (a), (b)에 해당되는 번호

와 Pa의 명칭이 순서대로 나열된 것은?

- ① 91, 231, Protactinium
② 89, 231, Pelactinium
③ 89, 231, Protactinium
④ 91, 231, Pelactinium



5. 어느 핵종의 동위원소가 그 핵종의 안정동위원소에 비해 훨씬 많은 양성자를 포함하고 있다면 이 동위원소에 대해 예상되는 가장 그럴듯한 붕괴는?

- ① β^- -붕괴 ② IT (Isomeric Transition) ③ β^+ -붕괴 ④ Mirror Nucleus

6. 핵의 질량을 von Weizsäcker의 semi-empirical mass formula의 논거에 따라 설명하고자 한다. 다음 중 핵자당 평균결합에너지(average binding energy per nucleon)가 핵의 질량수(A)에 의존하지 않는 효과는?

- ① volume effect ② surface effect ③ Coulomb effect ④ isospin symmetry effect

7. 중수핵(deuteron)은 $0.286 \text{ e} \cdot \text{fm}^2$ 의 quadrupole moment를 가진다. 이 사실과 관련되는 사항으로 연결시키기에 가장 부적합한 것은?

- ① Prolate 변형핵이다. ② L=2 상태(D-state)를 포함한다.
③ Tensor interaction이 작용한다. ④ 여기상태가 존재하지 않는다.

8. Yukawa의 중간자(meson) 이론으로써 성립된 중요한 개념의 하나로서는 '힘(가령 핵력)의 작용 반경(range)은 그 매개입자(field quantum)의 어떤 특성에 반비례한다'는 것이다. 여기서 어떤 특성에 해당되는 것은?

- ① 전하량 ② 정지질량 ③ 운동량 ④ 스핀

(계속)

$$E^2 = m^2 c^4 + p^2 c^2$$

$$\frac{dE}{dt} = \frac{d}{dt} \left(\sqrt{m^2 c^4 + p^2 c^2} \right)$$

$$\frac{dE}{dt} = \frac{1}{mc} \frac{d}{dt} \left(\frac{h}{\lambda} \right) = \frac{h}{m \lambda^2} \frac{d\lambda}{dt}$$

$$\Delta t \times c = \frac{ch}{\Delta E} = \frac{h}{m c^2}$$

9. Prove that for a spherically symmetric field $\phi(r)$, the second equality of

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r}$$

holds. (10 point)

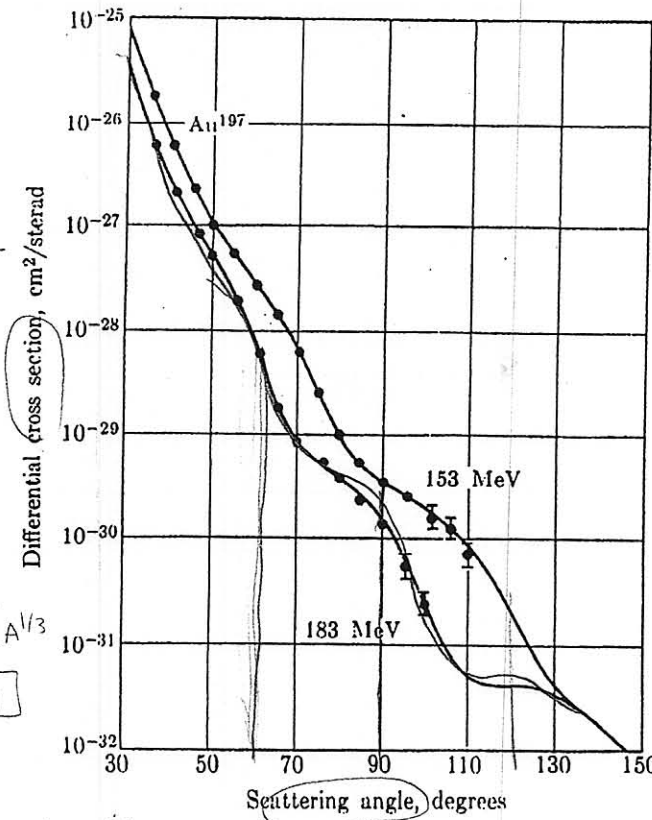
10. $^{64}_{29}\text{Cu}$ is experimentally 12.8 h of half-life, β^- 40%, β^+ 19%, EC 40% of the ratio of decay to daughter nucleus of the ground state of each conversion is known. (15 points)

a) For each decay mode, find the energy in MeV.

b) In addition to the above, 1% of $^{64}_{29}\text{Cu}$ decays emit a 1.34 MeV γ -ray. Explain the reason of the γ -ray. Consider the Q-value of the decay.

(In the graph, show the decay mode, energy, and units. Do not worry about the units in the problem, but use the information given. The mass of $^{64}_{29}\text{Cu}$ is 63.92977 u, $^{64}_{28}\text{Ni}$ is 63.92796 u, $^{64}_{30}\text{Zn}$ is 63.92914 u, electron mass is $0.511 \text{ MeV}/c^2$, $1 \text{ u} = 931.5 \text{ MeV}/c^2$.)

11. The Fraunhofer diffraction pattern behind a circular obstacle of diameter D is a series of dark rings with angular spacing $\Delta\theta \approx \lambda/D$ where λ is the wavelength of the incident wave. Using the de Broglie wavelength $\lambda = h/p$ for 183 MeV electrons, check whether this simple model applied to electron-nucleus scattering qualitatively predicts the angular distance between the wiggles in the attached figure. Explain the reason of the wiggles in the attached figure. Use that nuclear radius $R = 1.1 A^{1/3} \text{ fm}$, ($hc = 1239.7 \text{ MeV}\cdot\text{fm}$) and the 183 MeV electron can be treated relativistically. (10 point)



$$\frac{\Delta\theta}{2R}$$

$$pc^2 = \sqrt{E^2 - m_0^2 c^4}$$

$$\Delta\theta = \frac{\lambda}{2R}$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1239.7}{183} \approx 6.77 \text{ fm}$$

$$A = 197$$

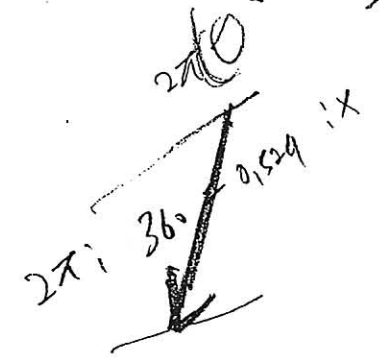
$$\Delta\theta = \frac{1239.7}{183 \times 2 \times 1.1 \times 197^{1/3}}$$

$$2\pi \approx 360$$

$$2\pi \approx 360$$

$$\Delta\theta \approx \frac{1}{1.1 A^{1/3}}$$

$$E^2 = m^2 c^4$$



12. Using the attached article and table, answer the following questions:

A fictitious nuclide has a $J = 1/2^+$ ground state and approximately evenly spaced excited states with $J = 9/2^+, 3/2^-, 1/2^+, 5/2^+, 7/2^-$ in order of ascending energy. Assume a level spacing of 1 MeV and mass number 153.

- a) Draw an energy level diagram for this nuclide and
 b) Indicate on the diagram drawn in a) the most likely gamma transition from each level (five lines in all).
 c) Write beside the lines their multipolarity. (E1, M2, etc.)

9-6 Selection Rules

The modes of electromagnetic radiation in the spherical cavity, referred to above, are quantized to $\Delta E = \hbar\omega$. Each mode is described by a vector function derived from the spherical harmonic functions $Y_{lm}(\theta, \phi)$. The multipole moments Q_{lm} and M_{lm} couple with the mode of order lm and with no others. Since one quantum in the mode lm carries an angular momentum $\sqrt{l(l+1)}\hbar$ and z -component $m\hbar$, radiation of multipolarity lm must remove from the nucleus a total angular momentum exactly equal to $\sqrt{l(l+1)}\hbar$, with z -component $m\hbar$, that is, the vector difference between the angular momenta J_a of the initial state and J_b of the final state must be $\sqrt{l(l+1)}$, and the z -component must change by $\Delta m_J = -m$. Reversing the argument, we find that the possible multiplicities of transitions between states J_a and J_b are given by

$$|J_a - J_b| \leq l \leq J_a + J_b \quad (9-26)$$

The parity of the nucleus may or may not change in a multipole transition, and the rules are simple enough to derive so that we can do it here. Consider the definition of the electric multipole matrix element (Eq. 9-24). The integral is a product of four functions, of which the first, (r^l) is always positive. The other three functions are either symmetric or antisymmetric, that is, their numerical values do not change by a reflection through the origin, but they may change sign. For the integral to give a result different from zero, the product of these three functions must be symmetric, because an antisymmetric function integrated over the whole nucleus yields as many negative contributions as positive. For even values of l , the spherical harmonic function is even; therefore the product of ψ_b and ψ_a must be even. For odd values of l , the product of ψ_b and ψ_a must be odd. We therefore get the following rule for the combination of parities of the initial and final state in an electric multipole transition:

$$\pi_a \pi_b = (-1)^l \quad (9-27)$$

By a similar argument, we find that for a magnetic multipole transition the relationship between the parities of the initial and final state is

$$\pi_a \pi_b = (-1)^{l+1} \quad (9-28)$$

The rules given above for the angular momentum and parity changes resulting from a transition of a given multipolarity are summarized in Table 9-1. These rules are called selection rules for gamma-ray transitions. Note that, in general, more than one type of multipole transition is possible between two states. For instance, if the angular momenta and parities of the initial and final states are $J_a = 1^+$ and $J_b = 2^-$, the possible l -values are 1, 2, and 3. Since the parity changes, the possible transitions are E1, M2, and E3. In general, however, the transition probability decreases very rapidly with increasing multipolarity l , so that for most practical purposes we can neglect all but the lowest possible

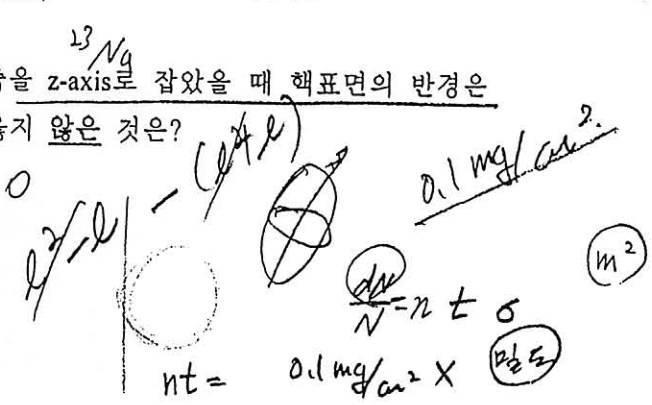
l -value. In the example above, therefore, the transition will be almost exclusively electric dipole radiation.

$0.1 \text{ mg/cm}^2 \times \frac{1}{23 \text{ g/mole}} \times 6.023 \times 10^{23} \text{ #/mole}$

9. 어떤 핵의 자체 좌표계 (body-fixed frame)에서 핵의 대칭축을 z-axis로 잡았을 때 핵표면의 반경은

$R(\theta, \phi) = R_0 [1 + \beta Y_{20}(\theta)]$ 로 기술된다. 다음 설명 중 옳지 않은 것은?

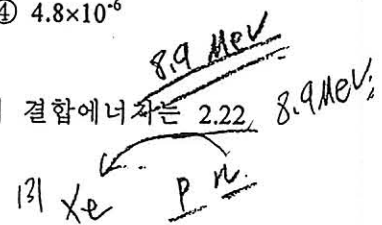
- ① 이 핵은 axi-symmetric quadrupole deformed nucleus 이다.
- ② 이 핵은 rotational model로 기술될 수 있다.
- ③ 이 핵은 prolate 핵이다.
- ④ 이 핵의 potential field는 변형되어(deformed) 있다.



10. 7 MeV 중양성자(deuteron)에 대해 $^{23}\text{Na}(d,p)^{24}\text{Na}$ 반응의 단면적은 $\sigma = 0.44 \text{ barn}$ 이다. 표적인 sodium target의 두께가 0.1 mg/cm^2 이라면, 표적에 입사하는 중양성자 중에서 ^{24}Na 를 형성하는 것의 비율로 가장 가까운 것은? 단, sodium은 100% ^{23}Na 이며, 원자량은 정수로 근사하라.

- ① 1.2×10^{-6}
- ② 2.4×10^{-6}
- ③ 3.6×10^{-6}
- ④ 4.8×10^{-6}

11. ^{132}Xe 에 대한 neutron separation energy는 8.9 MeV 이다. 또한 중양성자(deuteron)의 결합에너지는 2.22 MeV 이다. 이 수치를 이용하여 $^{131}\text{Xe}(d,p)^{132}\text{Xe}$ 반응의 Q-값을 구하시오.



12. $^9\text{Be}(p,n)^9\text{B}$ 반응의 Q-값은 $Q = -1.851 \text{ MeV}$ 이다. 이 반응이 일어나기 위해 laboratory 계에서 필요한 최소의 양성자 에너지 (threshold energy)는 몇 MeV인가? 답의 계산에는 근사적으로 정수질량을 활용하고 표적핵은 정지 상태로 간주한다.



13. Neutrons of mass m and kinetic energy T_1 are incident on a monoisotopic target of nuclear mass M . The neutrons are inelastically scattered from the target nuclei in such a way that they have zero energy in the laboratory system after the collision. Find the corresponding excitation energy E_{ex} for the state to which the target nuclei are being excited. Use non-relativistic formulas.

$$\frac{v'^2}{v^2} = \frac{1+\alpha}{2} + \frac{1-\alpha}{2} \cos \theta$$

14. Potential이 zero인 영역에서 positive energy E 의 양자계에 대한 Schrödinger equation의 radial part는 다음 식으로 주어진다.

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR_l}{dr} \right) + \frac{2m}{\hbar^2} \left[E - \frac{\ell(\ell+1)\hbar^2}{2mr^2} \right] R_l = 0 \quad (1)$$

여기서 radial wave function은

$$R_l(r) = j_l(kr) = \left(-\frac{r}{k} \right)^\ell \left(\frac{1}{r} \frac{d}{dr} \right)^\ell \left(\frac{\sin kr}{kr} \right), \quad k = \sqrt{2mE} / \hbar \quad (2)$$

으로 주어지는 spherical Bessel function $j_\ell(kr)$ 이다. (2)식으로부터 이것이 (1)식을 만족하는 해가 됨을 증명하시오. 단, (2)식의 미분연산자

$$\left(\frac{1}{r} \frac{d}{dr} \right)^\ell = \left(\frac{1}{r} \frac{d}{dr} \right) \left(\frac{1}{r} \frac{d}{dr} \right) \dots \left(\frac{1}{r} \frac{d}{dr} \right)$$

$$\frac{d^2}{dk^2} y + \frac{d}{dk} y + y = 0$$

는 $(1/r)(d/dr)$ 연산자가 ℓ 회 연속되는 경우로서 $(1/r^\ell)(d/dr)^\ell$ 로 간략하게 될 수는 없음을 참조하라.

(끝)