

수치해석 기초 중간고사 2007.10.24

Problem 1 (35)

1.1 Evaluate (i) $\frac{1-\cos x}{\sin^2 x}$ and (ii) $2 - \sin x - \cos x - e^{-x}$ for $x=10^{-5}$

1.2 Evaluate (iii) $e^{\frac{2}{x}} - \frac{x}{1+x}$ for $x=10^5$

1.3 What is the condition number of $\cos^{-1}x$? Where is it large?

What is the meaning of the large condition number ?

Problem 2 (35)

2.1 Calculate $f(-3.0)$ using (i) the first-, (ii) the second-, (iii) the third-, and (iv) the fourth-order interpolation polynomials based on the table below

2.2 Estimate the error of your calculations in 2.1

2.3 Can you compare the errors you obtained in 2.2 with the exact errors?

x_i	-7	-6	0	3	5
f_i	-23	-54	-954	-783	1

Problem 3 (30)

(3.1) Given (x_i, f_i) ($i=0,1,\dots,n$), derive the cubic spline as follows:

Set $F_i = f_i'$ for $i=0,1,\dots,n$, $h_i = x_{i+1} - x_i$, and obtain the triangular system of equations for F_i ($i=0,1,\dots,n$).

Hint: use piecewise Hermite interpolation defined below

(3.2) Calculate $f(1.5)$ using the cubic spline approximation from the table below

x_i	1.	2.	3.	4.
$f(x)$	1.	.25	.11111	.0625

i)				
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- The cubic Hermite interpolation for (x_k, f_k) and (x_{k+1}, f_{k+1}) for $(k=i, i+1)$ is given by

$$P_i(x) = f_i H_i(x) + f_{i+1} H_{i+1}(x) + f_i' \bar{H}_i(x) + f_{i+1}' \bar{H}_{i+1}(x) \text{ for } x \in [x_i, x_{i+1}]$$

$$H_i(x) = -2 \left(\frac{x_{i+1} - x}{h_i} \right)^3 + 3 \left(\frac{x_{i+1} - x}{h_i} \right)^2 \quad \text{and} \quad \bar{H}_i(x) = h_i \left[- \left(\frac{x_{i+1} - x}{h_i} \right)^3 + \left(\frac{x_{i+1} - x}{h_i} \right)^2 \right]$$

$$H_{i+1}(x) = -2 \left(\frac{x - x_i}{h_i} \right)^3 + 3 \left(\frac{x - x_i}{h_i} \right)^2 \quad \text{and} \quad \bar{H}_{i+1}(x) = h_i \left[\left(\frac{x - x_i}{h_i} \right)^3 - \left(\frac{x - x_i}{h_i} \right)^2 \right];$$

$$h_i = x_{i+1} - x_i$$