Elementary Numerical Analysis

2006년 1학기

Midterm Examination

April 26, 2006

1. Give the answer to the following questions. No need for derivation except for *f*. (20 points)
   a. The Lagrange interpolation polynomial of order n passing through $(n+1)$ given points.
   b. The error of an n-th order interpolation polynomial at an arbitrary point x in the interval in terms of a higher order derivative and polynomial.
   c. The martix resulting from the least square approximation of N data points using an n-th order polynomial.
   d. Roots of the n-th order Chebyshev polynomial.
   e. Finite difference approximation of the second order derivative for equally spaced three points and the dependence of its error on the interval size.
   f. Binary floating point number representation of 2.5 and 0.3125.

2. How can you solve an over-determined linear system conveniently? Explain that your solution is to minimize the square sum of each entry of the error vector. (10 points)

3. Explain the following as concisely as possible with only essential details. (15 points)
   1) The minimax property of the Chebyshev polynomial with the *two* interpretations.
   2) The way to determine the $(m+n+1)$ coefficients of the Pade approximation which is given by $f(x) \approx R_{m,n}(x) = \dfrac{\sum_{i=0}^{n} a_i x^i}{1+\sum_{i=1}^{m} b_i x^i}$.

4. Answer the following questions regarding the orthogonal polynomials which can be used in the least square approximation of functions at a given interval. (15 points)
   1) Derive the three term recursion relation to be used to generate a series of orthogonal polynomials.
   2) For an interval of [-1,1], derive orthogonal polynomials upto the 3-rd order using the above relation. You can use the Gram-Schmidt method if you prefer it. What is the special polynomial that these polynomials can represent? Name it.

5. Suppose that you established a differencing scheme to solve a boundary value problem, not knowing the order of error of the differencing scheme. Now you would like to estimate the true solution from a set of calculations employing different mesh sizes. Explain how you can do this by starting from the estimate of the error reduction factor for halving the mesh size. (15 points)

6. Answer the following questions regarding LU factorization. (15 points)
   1) Write a MATLAB script for LU factorization of a given Matrix A under the assumption that no pivoting is necessary during the elimination.
   2) If pivoting is necessary during the elimination, how should the algorithm be changed? You can explain with only words.

7. Explain rigorously that the following iterative scheme would converge
   $$Mx^{(k+1)} = b - Nx^{(k)}$$
   if the spectral radius of $M^{-1}N$ is less than 1.0 for a matrix splitting $A=M+N$. (10 points)