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문제 1.

(가) 10진수로 나타낸 실수 A=9875.36912745 를 2진수로 나타내시오.

$$\begin{array}{lll} & 2|9875 & \\ 2|4937 & \ldots 1 \\ 2|2468 & \ldots 1 \\ 2|1234 & \ldots 0 \\ 2|617 & \ldots 0 \\ 2|308 & \ldots 1 \\ 2|154 & \ldots 0 \\ 2|77 & \ldots 0 \\ 2|38 & \ldots 1 \\ 2|19 & \ldots 0 \\ 2|9 & \ldots 1 \\ 2|4 & \ldots 1 \\ 2|2 & \ldots 0 \\ 1 & \ldots 0 \end{array}$$

$$\begin{array}{r} 0.36912745 \\ \times \quad 2 \\ 0.73825490 \\ \times \quad 2 \\ 1.4765098 \\ \times \quad 2 \\ 0.9530196 \\ \times \quad 2 \\ 1.9060392 \\ \times \quad 2 \\ 1.8120784 \\ \times \quad 2 \\ 1.6241568 \\ \times \quad 2 \\ 1.2483136 \\ \times \quad 2 \\ 0.4966272 \\ \times \quad 2 \\ 0.9932544 \\ \times \quad 2 \\ 1.9865088 \end{array}$$

$$\therefore (10011010010011.0101111001)_2$$

$$or \quad 0.100110100100110101111001 \times 2^{14}$$

(나) 이 수를 다음 그림과 같이 전산기의 32 bit memory location 에 저장하고자 한다. Bit 내에 0 혹은 1을 채워 넣으시오.

지수부= $64+14=78$
$78=1001110_2$

1	1	0	0	1	1	1	0	1	0	0	1	1	0	1	0	0	1	0	0	1	1	0	1	0	1	1	1	1	0	0	1

(다) 32 bit memory에 저장된 A의 근사치를 fl(A) 라 한다. A-fl(A)를 구하라.

A=9875.36912745

$$fl(A)=2^{13}+2^{10}+2^{9}+2^{7}+2^{4}+2^{1}+2^{0}+2^{-2}+2^{-4}+2^{-5}+2^{-6}+2^{-7}+2^{-10}$$
$$=9875.368164$$

$$\therefore 0.00096345$$

(라) fl(A)의 유효 자리수는 몇 자리인가?

$$5\times10^{-8}\le\left|\frac{0.00096345}{9875.36912745}\right|\simeq9.75609\times10^{-8}\le5\times10^{-7}$$

=> 7자리

문제 2.

(가) 다음 관계식을 증명하시오.

$$f[x_i]=f(x_i)$$

$$f[x_i,x_{i+1}]=\frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i}=\frac{1}{h}\Delta f(x_i)$$

$$f[x_i,x_{i+1},x_{i+2}]=\frac{f[x_{i+1},x_{i+2}]-f[x_i,x_{i+1}]}{x_{i+2}-x_i}=\frac{1}{2h}\left[\frac{1}{h}\Delta f(x_{i+1})-\frac{1}{h}\Delta f(x_i)\right]$$

$$=\frac{1}{2h^2}\Delta^2 f(x_i)$$

$$f[x_i,x_{i+1},x_{i+2},x_{i+3}]=\frac{f[x_{i+1},x_{i+2},x_{i+3}]-f[x_i,x_{i+1},x_{i+2}]}{x_{i+3}-x_i}$$

$$=\frac{\frac{1}{2h^2}\Delta^2 f(x_{i+1})-\frac{1}{2h^2}\Delta^2 f(x_i)}{3h}=\frac{1}{3!h^3}\Delta^3 f(x_i)$$

$$\therefore f[x_i,x_{i+1},x_{i+2},\ldots,x_{i+h}]=\frac{1}{n!h^n}\Delta^n f(x_i)$$

(나) 다음 등 간격으로 주어진 (n+ 1) 개의 점에 대한 뉴튼 보간 다항식을 차분상을 써서 나타내어라. 또한 차분 연산자를 써서 나타내어라.

$$P_n(x)\simeq f[x_i]+f[x_i,x_{i+1}](x-x_i)+f[x_i,x_{i+1},x_{i+2}](x-x_i)(x-x_{i+1})$$
$$+\ \ldots\ +f[x_i,\ldots,x_{i+n}](x-x_i)\ldots(x-x_{i+n-1})$$

$$P_n(x)\simeq f(x_i)+\frac{1}{h}\Delta f(x_i)(x-x_i)+\frac{1}{2!h^2}\Delta^2 f(x_i)(x-x_i)(x-x_{i+1})$$
$$+\ldots+\frac{1}{n!h^n}\Delta^n f(x_i)\prod_{k=i}^{i+n-1}(x-x_k)$$

(다) 다음 표는 어떤 다항식 f(x)의 점들, 즉 (x,f(x))를 나타낸다. $\Delta^t f(x)(t=1,2,3,4,5\ldots)$를

구하고 표를 메워라. 이 다항식의 차수는? f(x)를 구하여라.

x	0	1	2	3	4	5	6	7
f(x)	0	-2	-8	0	64	250	648	1372
$\Delta f(x)$	-2	-6	8	64	186	398	724	
$\Delta^2 f(x)$	-4	14	56	122	212	326		
$\Delta^3 f(x)$	18	42	66	90	114			
$\Delta^4 f(x)$	24	24	24	24				
$\Delta^5 f(x)$	0	0	0					

$$P_n(x) \simeq f(0) + \Delta f(0)x + \frac{1}{2!}\Delta^2 f(0)x(x-1) + \frac{1}{3!}\Delta^3 f(0)x(x-1)(x-2) + \frac{1}{4!}\Delta^4 f(0)x(x-1)(x-2)(x-3)$$
$$= 0 - 2x - 2x(x-1) + 3x(x-1)(x-2) + x(x-1)(x-2)(x-3)$$
$$= x^4 - 3x^3$$

차수 : 4차

문제 3.
(가) 중앙 차분 연산자를 써서 다음 미분 공식을 유도하라.

$$\frac{df(x)}{dx} = \frac{2}{h}\sinh^{-1}\frac{\delta}{2}f(x)$$
$$= \frac{1}{h}(\delta - \frac{1}{24}\delta^3 + \dots)f(x)$$
$$\frac{d^2 f(x)}{dx^2} = \frac{1}{h^2}(\delta^2 - \frac{1}{12}\delta^4 + \frac{1}{90}\delta^6 \dots)f(x)$$
$$= \frac{\delta^2 f(x)}{h^2} - \frac{1}{12h^2}(h^4\frac{d^4 f}{dx^4}) + O(h^6)$$
$$= \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} - \frac{h^2}{12}f^{(4)}(x) + O(h^4)$$
$$(\text{단}, \delta^n = (h\frac{d}{dx} + \dots)^n)$$

(나) 전향 차분 연산자를 써서 다음 Simpson 적분 공식을 유도하시오

$$f(x)dx = \int_{x_{j-1}}^{x_j} f(x)dx + \int_{x_j}^{x_{j+1}} f(x)dx = (1+E)\int_{x_{j-1}}^{x_j} f(x)dx$$

$$\int_{x_{j-1}}^{x_{j+1}} = (1+1+\Delta)h\left(1+\frac{1}{2}\Delta - \frac{1}{12}\Delta^2 + \frac{1}{24}\Delta^3 + \ldots\right)f(x_{j-1})$$

$$= h\left(2+2\Delta+\frac{1}{3}\Delta^2 - \frac{1}{90}\Delta^4 + \ldots\right)f(x_{j-1})$$

$$= \frac{h}{3}\, f(x_{j-1}) + 4f(x_j) + f(x_{j+1}) - \frac{h^5}{90} f^{(4)}(x_j) + O(h^7)$$

(다) (나)의 Simpson 공식을 다음과 같이 하여 유도해 보아라.

$$I(x_{j+1}) = I(x_j+h) = I(x_j) + I'(x_j)h + \frac{h^2}{2} I'(x_j) + \frac{h^3}{3!} I''(x_j) + \frac{h^4}{4!} I^{(4)}(x_j) + \frac{h^5}{5!} I^{(5)}(x_j) + \frac{h^6}{6!} I^{(6)}(x_j) + O(h^7)$$

$$I(x_{j-1}) = I(x_j-h) = I(x_j) - I'(x_j)h + \frac{h^2}{2} I'(x_j) - \frac{h^3}{3!} I''(x_j) + \frac{h^4}{4!} I^{(4)}(x_j) - \frac{h^5}{5!} I^{(5)}(x_j) + \frac{h^6}{6!} I^{(6)}(x_j) + O(h^7)$$

$$I(x_{j+1}) - I(x_{j-1}) = 2f(x_j)h + \frac{h^3}{3} f'(x_j) + \frac{h^5}{60} f^{(4)} + O'(h^7)$$

$$= 2f(x_j)h + \frac{h^3}{3}\left[\frac{f(x_{j-1}) - 2f(x_j) + f(x_{j+1})}{h^2} - \frac{h^2}{12} f^{(4)} + O''(h^4)\right] + \frac{h^5}{60} f^{(4)} + O'(h^7)$$

$$= \frac{h}{3} f(x_{j-1}) + 4f(x_j) + f(x_{j+1}) - \frac{h^5}{90} f^{(4)}(x_j) + O(h^7)$$

문제 4.

(가)

$$h_0(X) = 3\left(\frac{X-X_1}{X_0-X_1}\right)^2 - 2\left(\frac{X-X_1}{X_0-X_1}\right)^3$$

$$\widetilde{h_0}(X) = \left[\left(\frac{X-X_1}{X_0-X_1}\right)^2 - \left(\frac{X-X_1}{X_0-X_1}\right)^3\right](X_1-X_0)$$

(나)

$$h_1(X) = 3\left(\frac{X-X_0}{X_1-X_0}\right)^2 - 2\left(\frac{X-X_0}{X_1-X_0}\right)^3$$

$$\widetilde{h_1}(X) = \left[-\left(\frac{X-X_0}{X_1-X_0}\right)^2 + \left(\frac{X-X_0}{X_1-X_0}\right)^3\right](X_1-X_0)$$