Solution to exam-4, April 27, 2004

Problem-1: 2 μCi of S^{35} is uniformly distributed in a testis. a) Find the absorbed dose rate of the testis in mrad/hour. b) Find the dose commitment to the testis in rad from the same source.

Data: radiological half-life of S-35 = 87.1 days

biological half-life of sulfur in the testis = 623 days

maximum energy of the beta-particle emitted by S^{35} = 0.1674 MeV/t

mass of a testis = 18 grams.

(Solution)

a) The average energy of S-35 beta-particle is approximated by:

$$\overline{E}(\beta) \approx \frac{1}{3} E^{\max}(\beta) = \frac{1}{3}(0.1674) = 0.0558 \quad \text{MeV}$$

Hence, since the emitted energy of beta-particles is 100% absorbed within the organ, the absorbed dose rate of beta particles in the testis is given by:

$$\dot{D}(\beta) = 1.6x10^{-13} q \frac{\overline{E}(\beta)}{M_T} \quad \text{Gy/sec}$$

where: $1.6x10^{-13}$ J/MeV

q = radioactivity, Bq or t/sec (transformations per sec)

$\overline{E}(\beta)$ = average energy of beta-particle per transformation, MeV/t

M_T = mass of organ, kg

$$\dot{D}(\beta) = (1.6x10^{-13} J/MeV)(2\mu Ci)(3.7x10^4 Bq/\mu Ci)\frac{0.0588 MeV/t}{18x10^{-3} kg}(10^5 mrad/Gy)(3600 \sec/hr)$$

= <u>13.9 mrad/hr</u>

b) The effective half-life of S-35 in the testis is:

$$T_E = \frac{T_R x T_B}{T_R + T_B} = \frac{(87.1)(623)}{87.1 + 623} = 76.4 \text{ days} = 1.834x10^3 \text{ hours} = 6.602x10^6 \text{ sec}$$

Hence, the effective elimination constant of S-35 in the testis is

$$\lambda_E = \frac{0.693}{T_E} = \frac{0.693}{1.834x10^3} = 3.78x10^{-4} \quad hr^{-1}$$

The dose commitment is defined by:

$$D_\infty \equiv \int_0^\infty \dot{D}(t)\,dt = \int_0^\infty \dot{D}_o e^{-\lambda_E t}\,dt = \frac{\dot{D}_o}{\lambda_E}$$

Hence, the dose commitment to the testis becomes:

$$D_\infty \equiv \frac{\dot{D}_o}{\lambda_E}$$

$$= \frac{13.2\ mrad/hr}{3.78x10^{-4}\ hr^{-1}}$$

= $3.49x10^4$ mrad

= <u>34.9 rad</u>

Problem-2: Find the average absorbed dose rate in μGy/sec to a testis (assuming a sphere of R=1.8 cm radius) by the uniformly distributed source Cs-137 of 1 μCi.

Data:

gamma emission by Cs-137: 0.662 MeV γ (85%) per nuclear transformation

At E_γ = 0.662 MeV; $(\mu)_{tissue}$ = 0.0327 cm^{-1} and $(\mu_e/\rho)_{tissue}$ = 0.0304 cm^2/g.

The average geometry factor for a sphere is given by: $\bar{g} = \frac{3\pi}{\mu}\left(1-e^{-\mu R}\right)$

(Solution)

The average geometry factor for a testis is:

$$\bar{g} = \frac{3\pi}{\mu}\left(1-e^{-\mu R}\right) = \frac{3\pi}{0.0327}\left(1-e^{-(0.0327)(1.8)}\right) = 16.47 \quad \text{cm}$$

The average absorbed dose rate of the organ is given by:

$$\bar{\dot{D}} = (1.6x10^{-13})\frac{C}{4\pi}\sum_i f_i E_i \left(\frac{\mu_e(E_i)}{\rho}\right)_m \bar{g}_i$$

where: $1.6x10^{-13}$ J/MeV

C = activity concentration in the organ = q/V, Bq/cm^3

q = 1 μCi = $3.7x10^4$ Bq

V = 4π(1.8 cm)^3/3 = 24.43 cm^3

$$= (1.6x10^{-13}\,J/MeV)\frac{3.7x10^4\,Bq/24.43\,cm^3}{4\pi}(0.85)(0.662MeV)(30.4\ cm^2/kg)(16.47\ cm)$$

$$= 5.43x10^{-9}\,Gy/\sec$$

$$= \underline{5.43x10^{-3}\,\mu Gy/\sec}$$

Problem-3: Following the instantaneous uptake of 10 kBq of I-131, a) Find the time-dependent activity (kBq) of I-131 in the thyroid assuming 20% of uptake directly goes to thyroid. b) Find the committed dose equivalent (mSv) to the thyroid. Neglect all other radiation sources except I-131 in the thyroid.

Data: thyroid weight = 20 grams

biological half-life of iodine in the thyroid = 138 days

radiological half-life = 8.05 days

iodine radiation energies (in MeV/t)

Beta radiations		Gamma radiations		
f_R	$\overline{E}_\beta^R$	f_R	E_γ^R	$AF(T \leftarrow T)_R$
0.016	0.0701	0.026	0.080	0.111
0.069	0.0955	0.054	0.284	0.120
0.005	0.1428	0.82	0.364	0.122
0.904	0.1917	0.068	0.637	0.124
0.006	0.285	0.016	0.723	0.120

(Solution)

a) The rate of change in the number of I-131 nuclides in the thyroid is give by:

$$\frac{dN_{thyroid;I-131}}{dt} = -\lambda_E(thyroid;I-131)N_{thyroid;I-131}\cdot$$

Solving the equation, the time-dependent I-131 activity in the thyroid becomes:

$$q_{thyroid;I-131}(t) = q_{thyroid;I-131}(0)\exp\left(-\lambda_E(thyroid;I-131)\ t\right)$$

The effective elimination constant of I-131 from the thyroid is:

$$\lambda_E(thyroid;I-131) = \frac{0.693}{T_E(thyroid;I-131)}$$

$$= 0.693\frac{T_R(I-131)+T_B(thyroid;iodine)}{T_R(I-131)xT_B(thyroid;iodine)} = 0.693\frac{8.05+138}{8.05x138}$$

$$= 9.11x10^{-2} \quad day^{-1}$$

$$= 1.1x10^{-6} \quad \sec^{-1}$$

Hence, the time-dependent I-131 activity in the thyroid following the instantaneous uptake of 10 kBq at t=0 is:

$$q_{thyroid;I-131}(t) = 2\exp\left(-(9.11x10^{-2})\ t\right)$$ kBq (where t is given in days.)

b) The ***committed dose equivalent*** is defined as the "dose equivalent to target tissue T accumulated over 50 years following the intake of nuclide j in source tissue S," as follows:

$$H_{50}(T \leftarrow S)_j \equiv \int_{t_o}^{t_o+50y} \dot{H}(T \leftarrow S)_j \, dt \qquad \text{Sv}$$

where,

t_o = time of intake

50 y = working life of 50 years.

Let, $$U_{sj} \equiv \int_{t_o}^{t_o+50y} q_j(t)\,dt \qquad \text{\# of transformations}$$

which is the "number of nuclear transformations of nuclide j occurred in S over 50 years following intake to the organ S at t = t_o."

Then, $$H_{50}(T \leftarrow S)_j = 1.6x10^{-10} U_{sj} SEE(T \leftarrow S)_j \qquad \text{Sv}$$

where: $$SEE(T \leftarrow S)_j = \frac{\sum_R f_R E_R AF(T \leftarrow S)_R w_R N_R}{M_T} \qquad \text{MeV/g.t}$$

Hence, the total committed dose equivalent to the target organ T from all the nuclides and all the source organs is

$$H_{50,T} = \sum_s \sum_j H_{50}(T \leftarrow S)_j \qquad \text{Sv}$$

In this case, the committed dose equivalent to the thyroid due to I-131 uptake is:

$H_{50}(thyroid \leftarrow thyroid)_{I\text{-}131} = 1.6x10^{-10}\ U_{thyroid;I\text{-}131}\ SEE(thyroid \leftarrow thyroid)_{I\text{-}131}$

$$U_{thyroid;I-131} \equiv \int_0^{50y} q_{thyroid;I-131}(t)dt$$

$$= \int_0^{50y} q_{thyroid;I-131}(0)e^{-\lambda_E(thyroid;I-131)t}dt$$

$$= \frac{q_{thyroid;I-131}(0)}{\lambda_E(thyroid;I-131)}$$

$$= \frac{(2\,kBq)(10^3\,tps)}{1.1x10^{-6}\sec^{-1}}$$

$$= 1.8x10^9 \qquad transformations$$

$$SEE(thyroid \leftarrow thyroid)_{I-131} = \frac{\sum_R f_R E_R AF(thyroid \leftarrow thyroid)_R w_R N_R}{M_{thyroid}} \qquad \text{MeV/g.t}$$

$$= \frac{\sum_R f_R E_R AF(thyroid \leftarrow thyroid)_R w_R N_R}{M_{thyroid}}$$

$$= \{(0.016)(0.0701)(1)(1) + (0.069)(0.0955)(1)(1) + (0.005)(0.1428)(1)(1) + (0.904)(0.1917)(1)(1)$$
$$+ (0.006)(0.285)(1)(1) + (0.0026)(0.080)(0.111)(1)(1) + (0.054)(0.284)(0.120)(1)(1)$$
$$+ (0.82)(0.364)(0.122)(1)(1) + (0.068)(0.637)(0.124)(1)(1) + (0.016)(0.723)(0.120)(1)(1)\} \times \frac{1}{20g}$$

= 0.0114 MeV/g.t

Hence, the committed dose equivalent to the thyroid due to I-131 becomes:

$H_{50}(thyroid \leftarrow thyroid)_{I-131} = 1.6x10^{-10}\ U_{thyroid;I-131}\ SEE(thyroid \leftarrow thyroid)_{I-131}$

= (1.6x10^{-10}) (1.8x10^{9} transformations) (0.0114 MeV/g.t))

= 3.28x10^{-3} Sv

= *3.28 mSv*

Problem-4: One of major thermal neutron interactions with the soft tissue in the human body is (n, γ) reaction of hydrogen; H-1 (n, γ) H-2. The accompanied γ-ray has the energy of 2.23 MeV. Find the dose-equivalent rate (mSv/hr) of a man of 70 kg who is exposed by the thermal neutron flux of $4\text{x}10^6$ n/sec.cm^2.

Data: # of hydrogen atoms per kg of tissue = $5.98\text{x}10^{25}$ #/kg

$\sigma_{n\gamma}$ = (n,γ) X-section of hydrogen for thermal neutrons = $3.3\text{x}10^{-25}$ cm^2

absorbed fraction of 2.23 MeV gamma within a total body = 0.278

(Solution)

For H-1 (n, γ) reaction; the specific activity of γ-emitter in the body is given by:

$q = \phi N \sigma_{n\gamma}$ Bq/kg

where, ϕ = thermal neutron flux, n/sec.cm^2

N = # of hydrogen atoms / kg of tissue, $5.98\text{x}10^{25}$ #/kg

$\sigma_{n\gamma}$ = (n, γ) X-section for hydrogen, $0.33\text{x}10^{-24}$ cm^2

q = ($4\text{x}10^6$ n/sec.cm^2)($5.98\text{x}10^{25}$ #/kg)($3.3\text{x}10^{-25}$ cm^2)

= $7.89\text{x}10^7$ Bq/kg

Hence, the absorbed dose rate due to 2.23 MeV photons is:

$$\dot{D}_{n\gamma} = q(Bq/kg)E(MeV)AF(tot \leftarrow tot)(1.6x10^{-13}J/MeV)$$

= *($7.89\text{x}10^7$ Bq/kg) (2.23 MeV) (0.278) ($1.6\text{x}10^{-13}$ J/MeV)*

= *$\underline{7.83\text{x}10^{-6}}$ $\underline{\text{Gy/sec}}$*

The dose-equivalent rate is:

$$\dot{H}_{n\gamma} = \dot{D}_{n\gamma} w_\gamma N_\gamma$$

= ($7.83\text{x}10^{-6}$ Gy/sec)(1)(1)

= $\underline{7.83\text{x}10^{-6} \text{ Sv/sec}}$

= 28.2 mSv/hr.