Problem 1 (30 points): Briefly explain the definitions of the following terms:

a. radiation	b. radioactivity
c. transformation constant	d. Avogadro's number
e. NORs	f. health physics

(Solution)

a. Radiation :

ⓐ 10CFR20 (Standards for Protection against Radiation)

alpha&beta particles, gamma rays, X-rays, neutrons, high-speed electrons, high-speed protons, other atomic particles capable of producing ion. (Radiation, as used in this regulation, does not include non-ionizing radiation, such as radio- or microwaves, or visible, infrared, or ultraviolet light.)

ⓑ 한국 원자력법 시행령

알파선, 중양자선, 양자선, 베타선, 기타 중하전입자선, 중성자선, 감마선 및 엑스선, 5 만 전자볼트 이상의 에너지를 가진 전자선

b. Radioactivity :

It means the emission of radiation by spontaneous nuclear transformation, or property of emitting radiation by spontaneous nuclear transformation.

c. Transformation constant (λ) :

The rate of spontaneous nuclear transformation of nuclide A is proportional to the number of nuclei A. $dN/dt=-\lambda N$(where, N=number of nuclei A, λ= Transformation constant) The unit of this constant is sec^{-1}, min^{-1} or hr^{-1}, etc. Similarly, the term of decay constant or disintegration constant is used instead of this constant.

d. Avogadro's number

= $6.02X10^{23}$ (number of atoms in a mole)

e. NORs

There exist naturally occurring radioactivities, which could be differentiated by the artificial (or man-made) radioactivities. Presently, 70 out of 340 naturally occurring nuclides are identified as radioactive. In particular, heavy elements(Z>82, Pb) are all radioactive.

(1) NOR from terrestrial origin : Major NOR nuclides from terrestrial origin are

① Uranium-, Thorium-, Actinium- and Neptunium- series

② Radon gases in air : Rn^{222}, Rn^{220}, Rn^{219} (Average concentration(air) = $2x10^{-6}$ Bq/㎖)

③ K^{40} : in crustal rock, ocean, human, urine etc (natural abundancy = 0.0119%)

(2) Cosmic rays :

① Composed of high energy charged particles : proton(85%), α-ray(14%), nuclei(1%).

② Very highly penetrating : Max(E)=1014eV & Average(E) = 1010eV

③ Interacting with air : e, n, γ, μ, π produced

④ varing in the radiation level by altitude, at poles, 11 year cycle solar flares.

(3) NOR produced by cosmic-rays : C^{14}, H^{3}, Be^{7}, Na^{22}, Na^{24}, Cl^{36}, etc.

f. Health Physics

The characteristics of radiation is: tasteless, odorless, senseless, invisible, and inaudible. The five senses of human are not able to detect it or measure its quantity. Without our knowledge, it could present us great biological effects. Hence, “***health physics”*** (sometimes called “***radiation physics”***.) is the arts and science of protecting the public health and conserving the environment against radiation. Meanwhile, radiation is beneficially used for medical, industrial and agricultural purposes. The beneficial use of radiation and radioisotopes could be also included in health physics.

Problem-2 (20 points): a) Compute the number of grams and the number of radioactive atoms contained in 1 mCi of Ra-226 ($T_{1/2}$ = 1620 y). b) What will be the activity of Ra-226 in MBq after 2000 years later?

(Solution)

a) The activity is defined by: $A = N \lambda$.

Hence, $N = A / \lambda = A (T_{1/2}/0.693)$.

N = (10^{-3} Ci) ($3.7\text{x}10^{10}$ tps/Ci) {(1620 years)(365 days/year) (24 hours/day) (3600 seconds/hour) / 0.693} = $2.727\text{x}10^{18}$ atoms or

m = {($2.727\text{x}10^{18}$ atoms)/($6.02\text{x}10^{23}$ atoms/mole)} (226 grams/mole)

= $1.02\ \text{x}10^{-3}$ grams = 1.02 mg

b) The rate of change in the number of Ra-226 nuclei is:

$dN_A/dt = -\lambda_A N_A$ where λ_A = 0.693/1620 years = $4.28\text{x}10^{-4}$ year^{-1}.

dy/dx + a(x) y = h(x): Solution y(x) = 1/p ∫ p h(x) dx + c/p; where, c= constant and p = exp(∫ a(x) dx) = integrating factor.

Solving the differential equation,

$dN_A/dt + \lambda_A N_A = 0$ $y= N_A(t)$ $a(x)= \lambda_A$ $h(x)= 0$

Integrating factor: $p = \exp(\int a(x)\, dx) = \exp(\int \lambda_A\, dt) = \exp(\lambda_A t)$

$N_A(t) = c \exp(-\lambda_A t)$

At t=0, $N_A(t) = N_A(0)$, then $c = N_A(0)$. $\therefore N_A(t) = N_A(0) \exp(-\lambda_A t)$

Hence, at t = 2000 years,

A_A(t=2000 years) = (10^{-3} Ci) ($3.7\text{x}10^{4}$ MBq/Ci) exp($-4.2727\text{x}10^{-4}$ year^{-1} x 2000 years) = $1.57\text{x}10^{4}$ kBq

Problem-3 (10 points): Find the specific activity (Bq/kg) of K-40 in the human body.

Data: amount of potassium in human body = 1.7 g/kg

Natural abundance of K-40 (K-40/K) = 0.0119%
Half-life of K-40 = 1.3×10^9 yr

(Solution)

$$\lambda \, of \, K^{40} = \frac{\ln 2}{1.3 \times 10^9 \times 365 \times 24 \times 3600 \sec} = 1.69 \times 10^{-17} \sec^{-1}$$

$$A(Bq/kg) = N(Number \, of \, K \, atoms/kg)\,\lambda(Transfermation \, constant, \sec^{-1})$$

$$= (1.7 g/kg)(1.19 \times 10^{-4} \frac{K^{40}}{K})(\frac{6.02 \times 10^{23} \, atoms/mol}{40 g/mol})(1.69 \times 10^{-17})$$

= 51.5 Bq/kg

Problem-4 (20 points): Tc-99m (half-life = 6.00 hours) is usually used for liver-scanning for a medical diagnostic purpose. Tc-99m is generated as the daughter product of Mo-99 (half-life = 66.0 hours) following its beta transformation. If 1.0 Mbq Tc-99m is needed for a liver-scanning, and if 2 days elapse between shipment of the Tc-generator (Mo-99) and the use of Tc-99m in the test, how many micro-curie of Mo-99 must be shipped?

(Solution)

Mo-99 (beta-transformation with $T_{1/2}$ = 66 hours) → Tc-99m ($T_{1/2}$ = 6 hours) →

The rate of change in the number of Mo-99 nuclei during shipping is:

$dN_A/dt = -\lambda_A N_A$ (A)

The rate of change in the number of Tc-99m nuclei during shipping is:

$dN_B/dt = \lambda_A N_A - \lambda_B N_B$ (B)

Solving Eq. (A),

$dN_A/dt + \lambda_A N_A = 0$

Integrating factor: $p = \exp(\int \lambda_A\, dt) = \exp(\lambda_A t)$

$N_A(t) = c \exp(-\lambda_A t)$

At t=0, $N_A(t) = N_A(0)$, hence $N_A(0) = c$.

$\therefore N_A(t) = N_A(0) \exp(-\lambda_A t)$

Inserting this equation into Eq. (B),

$dN_B/dt + \lambda_B N_B = \lambda_A N_A = \lambda_A N_A(0) \exp(-\lambda_A t)$

Integrating factor: $p = \exp(\int \lambda_B\, dt) = \exp(\lambda_B t)$

$$N_B(t) = \exp(-\lambda_B t) \int \exp(\lambda_B t)\, \lambda_A N_A(0) \exp(-\lambda_A t)\, dt + c \exp(-\lambda_B t)$$

$$= \lambda_A N_A(0) \exp(-\lambda_A)t / (\lambda_B-\lambda_A) + c \exp(-\lambda_B t)$$

At t=0, $N_B(0) = 0$, then

$0 = \lambda_A N_A(0) / (\lambda_B-\lambda_A) + c$ $\quad\quad$ $c = - \lambda_A N_A(0) / (\lambda_B-\lambda_A)$

$$\therefore N_B(t) = \lambda_A N_A(0) \exp(-\lambda_A t) [1 - \exp\{-(\lambda_B-\lambda_A) t\}] / (\lambda_B-\lambda_A)$$

In activity expression,

$$A_B(t) = N_B(t) \lambda_B = \lambda_B A_A(0) \exp(-\lambda_A t) [1 - \exp\{-(\lambda_B-\lambda_A) t\}] / (\lambda_B-\lambda_A)$$

Now, $\lambda_A = 0.693/(66 \text{ hours}) = 1.05\text{x}10^{-2} \text{ hour}^{-1}$.

$\lambda_B = 0.693/(6 \text{ hours}) = 1.155\text{x}10^{-1} \text{ hour}^{-1}$

After 1 day cool, $A_B(t\text{=1day}) = 1$ MBq, hence

$1 \text{ MBq} = (1.155\text{x}10^{-1} \text{ hour}^{-1}) A_A(0) \exp(-1.05\text{x}10^{-2} \text{ hour}^{-1} \text{ x } 24 \text{ hours}) [1 - \exp\{-(1.155\text{x}10^{-1} - 1.05\text{x}10^{-2}) \text{ hour}^{-1} \text{ x } 24 \text{ hours}\}] / (1.155\text{x}10^{-1} - 1.05\text{x}10^{-2}) \text{ hour}^{-1}$

$\therefore A_A(0) = 1.272$ MBq

Problem-5 (20 points): a) Find the amount of Cs-137 radioactivity (in curies) in the core of APR1400 (3,800 MWth) at the end of 1-year full-power operation. The half-life of Cs-137 is 30.0 years and fission yield is 6.18%. Assume 1 fission produces 3 x10^{-11} Joules.

b) Find the amount of I-133 radioactivity (in curies) in the equilibrium core of APR1400 (3,800 MWth) at the end of 1-year full-power operation. The half-life of I-133 is 20.3 hours and fission yield is 6.69%.

(Solution)

The rate of change in the number of nuclide *i* at $\underline{r}$ in the reactor core is given by:

$$\frac{dN_i(\underline{r},t)}{dt} = \gamma_i \int_E dE\Sigma_f(\underline{r},E)\phi(\underline{r},E) - \lambda_i N_i(\underline{r},t)$$

Solving the equation with the integrating factor: p = exp(∫ λ_i dt) = exp(λ_i t), the number of nuclide *i* becomes:

$$N_i(\underline{r},t) = \frac{\gamma_i \int_E dE\Sigma_f(\underline{r},E)\phi(\underline{r},E)}{\lambda_i}\left(1-\exp(-\lambda_i t)\right)$$

Hence, by integrating over the core volume, the total activity in the core becomes:

$$A_i(t) = \int_V d\underline{r}\lambda_i N_i(\underline{r},t) = \gamma_i \int_V d\underline{r}\int_E dE\Sigma_f(\underline{r},E)\phi(\underline{r},E)\left(1-\exp(-\lambda_i t)\right)$$

where,

$\int_V d\underline{r}\int_E dE\Sigma_f(\underline{r},E)\phi(\underline{r},E)$ is the total fission rate in the reactor.

Hence, for a 3,000 MWth reactor, the total fission rate becomes:

$\int_V d\underline{r}\int_E dE\Sigma_f(\underline{r},E)\phi(\underline{r},E)$ = (3,000 MWth)(10^6 W/MW)(1 fission per 3x10^{-11} Joules) = 10^{20} fissions/sec

The activity becomes:

$$A_i(t) = \gamma_i (10^{20})\left(1 - \exp(-\lambda_i t)\right) \quad \text{Bq}$$

a) In case of Cs-137; at the end of 2-year full power operation, the activity becomes: (λ_i = 0.693/30.0 = 0.0231 year^{-1})

$$A_i(t) = \gamma_i (10^{20})\left(1 - \exp(-\lambda_i t)\right) = (0.0618)(10^{20})(1 - \exp(-(0.0231)(2))$$

= 2.79x10^{17} Bq

= (2.79x10^{17} Bq)/(3.7x10^{10} Bq/Ci) = 7.54x10^{6} Ci

b) In case of I-133; at the end of 1-year full power operation, the activity becomes saturated: (λ_i = 0.693/20.3 = 0.0341 hour^{-1} = 2987 year^{-1})

$$A_i(t) = \gamma_i (10^{20})\left(1 - \exp(-\lambda_i t)\right) = \gamma_i (10^{20}) = (0.0669)(10^{20})$$

= 6.69x10^{18} Bq

= (6.69x10^{18} Bq)/(3.7x10^{10} Bq/Ci) = 1.8x10^{8} Ci